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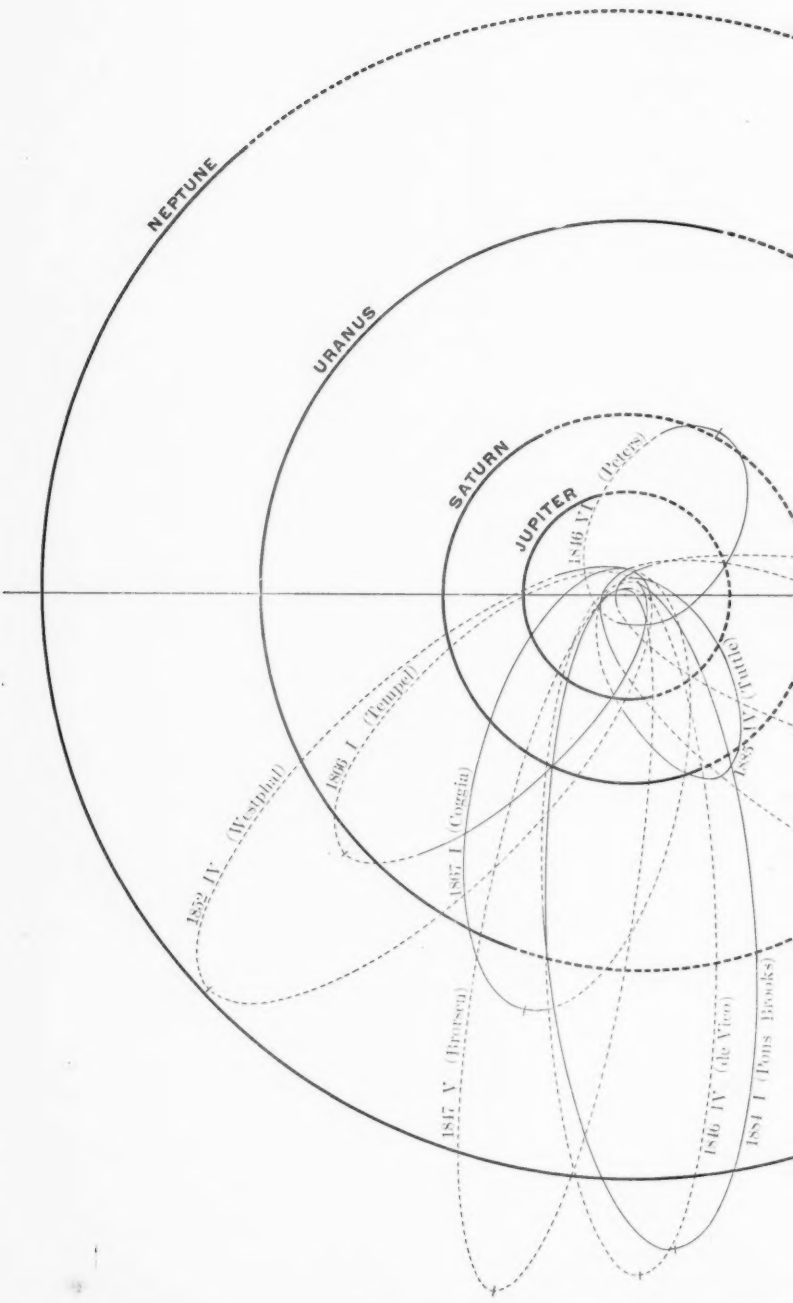
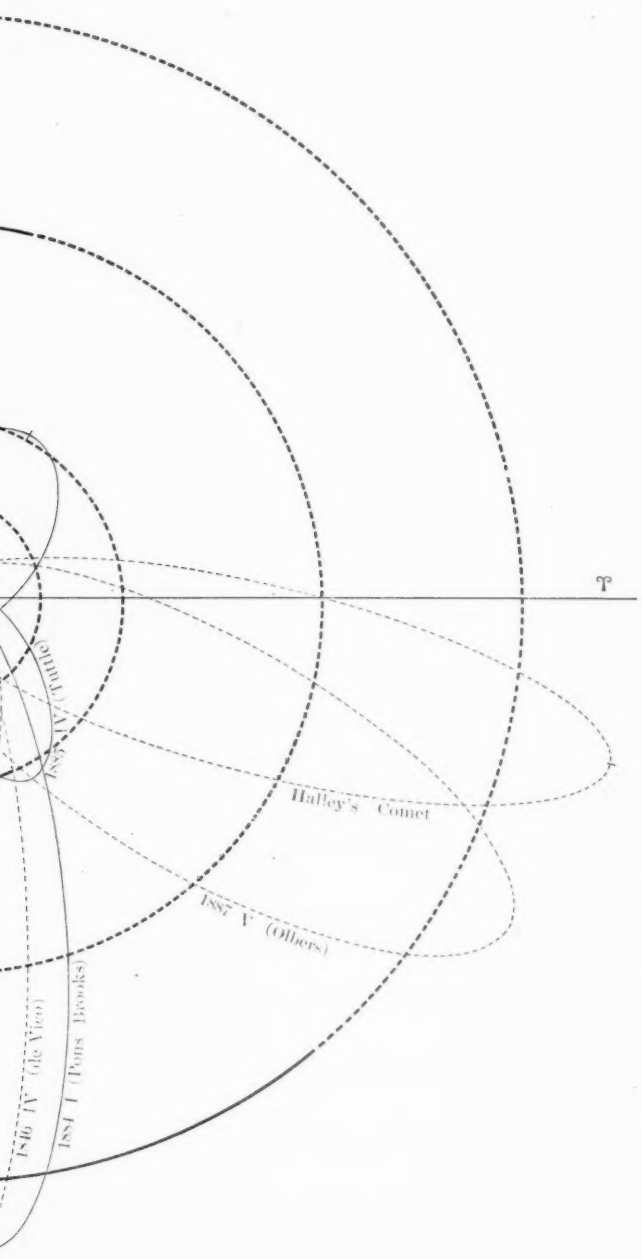
PLATE I.



COMET FAMILIES OF SATURN, URANUS

I.



, URANUS AND NEPTUNE.

DRAWN BY DR. A. G. SIVASLIAN, ANATOLIA COLLEGE, MARSOVAN, TURKEY.

Popular Astronomy.

Vol. VI. No. 1. MARCH, 1898. Whole No. 51.

COMETS AND COMET FAMILIES.

W. W. PAYNE.

The re-appearance of Winnecke's comet which was first seen at Lick Observatory Jan. 2, 1898, by Mr. Perrine calls attention again to Jupiter's family of comets, and especially to this member of the family which was formerly rated as third, because its period of revolution around the Sun was supposed to be only 5.58 years, the two other periodic comets having shorter times of revolution being Encke's and Tempel's, (1873 II) whose respective times of revolution are 3.29 years and 5.15 years. Since 1891 Swift's comet (1880 V) whose period was supposed to be 6 years has been reduced to 5.50 years, and Tempel's first (1867 II) which was 5.98 is now probably 6.51 years. These changes in data respecting the orbits of the inner members of the Jupiter family were all made in the excellent drawing of the orbits of the numerous family belonging to Jupiter which was published in October, 1893, and the plate will be found in number 2, of POPULAR ASTRONOMY facing page 64. Referring to that figure the Earth's path is at once noticed, as the small heavy circle near the center. The perihelion point which the Earth passes in its orbit about Jan. 1, each year, is ten degrees to the left of a point directly above the Sun and nearest to the top of the chart. The Earth then was at this point in its path when the Winnecke comet was first seen, at this return on Jan. 2, at the Lick Observatory. The horizontal line in the chart through the Sun is the line of the equinoxes, the vernal equinox being to the right. Now it is easy for any one to understand how the astronomer says the perihelion point of the Earth's orbit is 100° 21′ 22″ in longitude if the reckoning begin at the horizontal line, and be counted from left to right above to the point named for the beginning of January.

In the same easy way we can locate the comet at the time it was found Jan. 2. It is known to most readers probably that all elliptical comets of periods less than 100 years, except Halley's comet, revolve in their orbits in the same direction as the planets do. To illustrate this, imagine a watch to be placed at the point representing the Sun in the chart with the face up, and the hands

moving in the plane of the Earth's path. As the observer looks down on the watch he will see the hands moving in a direction opposite to that of the comets and planets. Hence all of Jupiter's family of comets revolve around the Sun in the same direction as the planets do which is said to be counter clockwise in direction.

To find the comet's place in its path we have only to refer to tables that are now being published from month to month in this and other similar journals. From such tables we notice that the comet was about 160 millions of miles from the Earth when first seen at Lick Observatory in January last. With the radius of the Earth's orbit as 93 millions of miles, any one can measure from the Earth's position in its orbit to the left and determine the point approximately on the comet's orbit which will indicate its place when re-discovered at this return. For January and February it will be easily seen that the comet and the Earth will be slowly approaching each other, but after the first of March the two bodies will begin to separate, because the comet will have crossed the Earth's orbit, in front of our planet, passing nearer the Sun than the path of the Earth, until on March 21 it reaches the perihelion point, and then it will turn in its journey again into space for another periodical absence of six years from our celestial neighborhood.

We have been thus particular about a few of the many common data that might be mentioned about this, or almost any other short-period comet, to interest unscientific readers in the meaning and use of comet charts. If careful attention is given to them they may be easily understood, and they will bring much help in getting definite ideas about the motions and the behavior of these erratic celestial visitors.

The history of the Winnecke comet is worthy of a brief, passing notice. It was discovered by M. Pons, June 12, 1819, and the distinguished Encke found for it a period of revolution around the Sun of 5½ years which is a little less than the period given earlier. The next time the comet appeared was not until March 8, 1858, when it was found by Winnecke of Bonn, and it was then regarded by him as a new comet, although soon after he identified it as the Pons comet. Its return in 1863, was not seen probably because of its unfavorable place for observation. In June, 1869, Winnecke saw it April 9, and described it as a faint object not less than 6′ or 8′ in diameter. This comet was again visible in March, 1875, and from some calculations made at this time, it has been thought that it might be identical with a comet discovered by Pons in 1808. This comet was due again in 1880,

but it was not seen. It was observed in 1886 in the southern hemisphere. It passed its nearest point to the Sun at this return 12 days sooner than the predicted time, and this fact led Oppolzer to think that its movements could not be explained by the theory of gravitation alone, and so the existence of some resisting medium in its path seemed to be indicated. At the last return of the comet in 1892, it was observed from April to November, at various places in the United States and elsewhere.

The question of a resisting medium in space is one full of interest to the physicist and to the mathematician. It was long ago suggested as a reason why the motion of the Encke comet did not conform to the law of gravitation as understood by Newton and other scholars since his time. Its period was about 2½ hours shorter than that determined by computation from all known data down to 1868. From computations made at this time, it appears that this error is reduced about one-half, so that it is possible and to some degree probable, that the small error of period in the Encke comet and that of the Winnecke comet will yet be explained without the hypothesis of a resisting medium in space. So far as we know investigation in any direction during the last ten years, has not contributed useful information in determining the properties of a resisting medium in space if such a thing exists. That there is a medium in space for the transmission of light and other cosmic forces seems a necessity, but whether it does really resist the movement of celestial bodies in space to any appreciable degree is a problem which the scholars in science have not yet solved.

But the main purpose of this article is to present a chart showing the other comet families belonging to the planets Saturn, Uranus and Neptune. The accompanying plate was drawn by Dr. A. G. Sivaslian, the first graduate of the post course in Mathematics and Astronomy at Carleton College, who is now at Anatolia College, Marsovan, Turkey, and in charge of the Department of Mathematics and Astronomy in that Institution. The drawing was made some time ago, but was delayed on its way in reaching us, so that an interval of more than four years has elapsed since the publication of the chart of the Jupiter family of comets to which this drawing is intended to be a companion. In passing it should be noted, that a symbol representing the Sun at the center of the planet orbits should have been inserted to make its appearance more complete. That was accidentally overlooked in preparing the copy, but it can be readily imagined and located by the reader.

From this drawing it will at once be seen that the Saturn family consists of two members: Comet 1846 VI (Peters) and Comet 1885 IV (Tuttle) the most distant points in their orbits from the Sun being only a little way beyond the path of Saturn. Not much is known of the first comet. Its discoverer was C. H. Peters who found it in June, 1846. He also computed its orbit whose period is 12.8 years with an uncertainty of one whole year. At the time of discovery the comet was visible for four weeks.

Comet 1885 IV (Tuttle) received its name from the discoverer, H. P. Tuttle, who found it Jan. 4, 1858, while at Harvard College Observatory. (See article by Mr. Tuttle elsewhere in this number). Its orbit was computed by Bruhns and a period of 13.6 years was found as a result. Its last appearance was in 1885 and was first seen in that year by Perrotin, orbit recomputed by Rahtz, the comet being visible for a period of five weeks, during the months of August and September. If the period is correct this comet ought to return in 1899 about the middle of May if there has been no disturbance in its path since 1885. It is to be hoped that careful watch will be made for its reappearance. It is also noticeable that the aphelion points of the two members of this family are about 120° apart, a very considerable distance seeing there are but two. In Jupiter's family the aphelion points of all orbits but 6 lie within about the same distance on the planet's path. If these planets have been two thousand years capturing their respective families from the vagabond wanderers of space, the vernal equinox has moved to the westward in that period about 30°. So far as the Jupiter family is concerned, if the vernal equinox were moved eastward 15°, the autumnal equinox would be near the middle of the aphelion points of nearly all the family. This would mean a period of activity in the capture theory of something more than 1000 years if nothing more be said of the possibilities of planetary control.

On the other hand when we look at the position of the Saturn family, it is almost exactly reversed from that of the Jupiter family. However, the number in the Saturn family is so small, that little if anything can be inferred respecting relations to suggest a possible cause for the positions of the orbits of this family.

The comets which belong to the family of Uranus are two and possibly one more. We have presented two in the accompanying chart; the comet of 1866 I (Tempel) and comet 1867 I. The distances which the comets of the Uranus family reach, in out-

ward motions from the Sun, range between 18.3 and 20.1, the unit of these members being the Earth's mean distance from the Sun, or about 93,000,000 of miles. The Tempel comet's path has an aphelion distance of 19.7, while that of the 1867 I is 19.3. The time of revolution around the Sun assigned to the former is about 33 years, that of the latter about 40 years. The study of the comet of Tempel has some points of interest, since 1866, on account of its connection with the stream of November meteors. In 1864 Professor H. A. Newton of Yale College became much interested in the November meteors and made a study of the periods of marked displays. He determined orbits for the November Leonids, one of which Professor Adams, of Cambridge, England, favored strongly as the true one for this notable stream whose period was 33¼ years. In 1866 a grand shower of meteors occurred, as was predicted, and the livliest attention of astronomers everywhere was drawn to this new field of astronomical investigation. Soon after Leverrier of France published the orbit of the Leonid meteors obtained from the observed position of the radiant and the periodic time assigned by Adams, and independently and almost simultaneously Oppolzer published his orbit of the comet of Tempel, 1866, and the two orbits were found to be practically identical. About the same time Schiaparelli of Milan had found a similar relation to exist between the Perseid stream of meteors and Tuttle's comet of 1862.

Such new and astonishing facts concerning the relation of comets and meteors in some instances, at least, gave a new impetus to the study of meteors. The fact that two celestial bodies, so unlike in kind, as previously supposed, should move in the same paths through space, and obey the law of gravitation in every way alike, at once awakened an interest in meteors, meteoric showers and meteoric trains before unknown. Hence for 30 years astronomers have been looking forward to the time when the next great display of the Leonid shower shall occur. The date set for it is close at hand and will take place in 1899, if the period has been as carefully determined as skilled observers generally believe. More information concerning the nature of the meteors of this memorable stream may be learned if the next display can be favorably observed, in the study of their spectra and in other possible ways to be devised by the ingenuity of modern observers.

We have not space, this time, to speak of the remaining member or members of this family, or of the family of six comets belonging to Neptune. We ought also, at another time, to speak of

that family of comets which some think belongs to a trans-Neptunian planet, and answer the question if possible, whether their aphelion points give any suggestion as to the distance of that unknown, far-off planet which may be the disturber of Neptune's motion that so far astronomers are unable to find, although some careful work has been given to the problem in various ways. Our readers ought also to notice that the families of most of the outer planets do, in the main, pretty plainly show that the principle of "capture" is undoubtedly the thing that governs in bringing the comets into the solar system, and in placing them all probably into families with suitable revolution periods dependent on definite relations that the planets and the Sun have sustained to the comets in long or short periods of time in the past. The nice adjustment of the motion of the planets in their orbits, as related to the velocity of comets coming toward our solar system from the depths of space is a point worthy of notice; but even beyond this, the motion of the Sun with the solar system in space affords the strongest support for the theory of "capture" in the outer families as well as in the family of Jupiter already noticed in a previous article.

February, 1898,
Northfield, Minn.

THE LOST PERIODIC COMET (TUTTLE-SCHULHOF) III 1858.

H. P. TUTTLE.

FOR POPULAR ASTRONOMY.

Early in the evening of May 2d, 1858, while searching for comets at the Harvard College Observatory, I detected in the constellation, *Leo Minor*, an exceedingly faint wisp of light which, although on the border of a region rich in faint nebulæ I felt sure was a comet. Professor G. P. Bond, who was at that time using the great equatorial, immediately turned that instrument upon the object and his well-trained eye pronounced it to be a comet instantly. Of the twelve telescopic comets which I have discovered between August, 1857 and December, 1888, this one was by far the faintest, in fact it could only be seen in my comet-seeker of four inches, by *indirect* vision.

I have not the references at hand, but I think the comet was not observed at Cambridge, after the middle of May. It was the intention of Professor Bond again to make additional observations early in June, but bad weather set in and continued during

the greater part of the month of June. The late James Ferguson, of the Naval Observatory, informed me that, armed with an ephemeris, he was unable to pick it up with the Washington equatorial. At Ann Arbor, Professor J. C. Watson succeeded in making only one observation, and this on May 30th or 31st.

This comet, whether looked for or not, was not seen at any of the European Observatories.

In January, 1864, Professor Watson sent me his observation of May 30th or 31st, and with others made at Cambridge I computed an orbit which was almost identical with those computed by Professors Hall and Safford in 1858, from the Cambridge observations alone.

Many years ago the distinguished French astronomer, Mr. L. Schulhof, investigated the orbit of this comet and found that it was moving in an orbit of about six years and seven months. His investigations and a sweeping ephemeris were published in the *Astronomische Nachrichten*, the No. or Nos. of which I do not now remember. Dr. Spitaler of Vienna appears to have been the only astronomer who made use of Mr. Schulhof's ephemeris. With the great Vienna refractor he found an object near the computed place which a few nights later had disappeared. Unfavorable weather compelled him to give up further search for the missing body.

Dr. J. R. Hind, late Sup't of the British "Nautical Almanac," gives an interesting account of this comet in *Nature* of some fifteen years ago.

Mr. Schulhof deserves great credit for his investigations, and some future astronomer with the zeal and perseverence of a Barnard, may yet pick it up with his camera somewhere "on the confines of creation."

January, 1898,

Brightwood, D. C.

LIGHT FLUCTUATIONS OF VARIABLES AS RELATED TO CHANGES IN POSITIONS OF PERIASTRON AND IN ORBITAL ECCENTRICITY.

G. W. MYERS.

For Popular Astronomy.

The increasing number of variable stars whose light changes are finding at least provisional explanation on the hypothesis of the existence in their systems of unequally illuminated components adds interest to questions relating to the effects on light

variation of perturbations such as the orbits of secondaries are likely to undergo. In the face of well established facts, it can hardly be supposed that the secondaries, whose eclipses and occultations are mainly concerned in the light fluctuations of these stars, are the only bodies involved. The analogies furnished by multiple stellar systems and by our own planetary system, to say nothing of Dr. See's convincing arguments for the actual existence in the system of 70 Ophiuchi of an invisible disturber, heighten the suspicion of the presence in certain variable systems of numerous unseen attendants, almost to a conviction. That such bodies come within the reach of neither telescopic nor spectroscopic vision in no way militates against their acceptance, since low brightness coupled with high orbital inclination to the sight line would remove them wholly from vision of either sort. If we add to the evidence recited above, the testimony of the faint companions of Sirius and Procyon and the invisible attendants of Algol, η Aquilae and others, it would even seem that the burden of proof of the existence or non-existence of the conditions necessary to orbital perturbations in variable star systems now lies with the negative.

Assuming then, that in some of these systems such conditions do exist, it will be interesting to draw a few inferences. Many of these inferences are sufficiently elementary and interesting to justify their insertion here. The writer cannot but think that the ideas set forth below have occurred to many students of variable stars, though he has been unable to find anything similar in the astronomical journals available to him. It is barely possible that others may be in the same condition as the writer, and if so, it is to such he would direct his remarks.

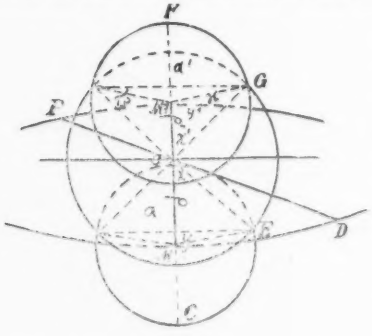


Fig. 1.

The specific subject of the following inquiry is whether or how the nature of the light changes of variables to which the so-called satellite theory is applicable, is affected by a revolution of the line of apsides and a change of eccentricity of the relative orbit.

Let the positions of the satellite at the instants of superior and inferior conjunctions be represented by the circles whose centres are at M_1 and M respectively, and let the larger

circle represent the position of the primary. The alternate passage of the smaller body before and behind the larger produces the light variation. The bodies are supposed to be unequally bright and the discs to be circular. The arcs PM_1 and DM represent portions of the orbit of the secondary.

Let α and α' denote the areas common to both discs at the times of superior and inferior conjunctions respectively.

r and r', the radii vectores at the same instants, ρ and ρ', the radii vectores of the apparent orbit at these epochs, $\varkappa$, the ratio of the radius of the secondary to that of the primary, λ, the ratio of the brightnesses in the same sense.

$\tilde{\omega}$, the true anomaly of the secondary at inferior conjunction,

x, y, x' and y', the angles EOC, EMO, GOF and $GM'O$ in the figure.

σ, the common angle of projection of r and r' and

L, the ratio of the brightnesses at the conjunctions.

We have them directly from the figure,

$$(1) \qquad L = \frac{1+(1-\alpha)\,\lambda\varkappa^2}{1+(\lambda-\alpha')\,\varkappa^2} = \frac{\text{Brightness at the minimum of superior conjunction}}{\text{Brightness at the minimum of inferior conjunction}}.$$

The denominator may also be written $1+\left(1-\frac{\alpha'}{\lambda}\right)\lambda\varkappa^2$ for easier comparison with the numerator. If now,

$\lambda < \frac{\alpha'}{\alpha}$, there will be two unequal minima, the lower falling at inferior conjunction.

$\lambda > \frac{\alpha'}{\alpha}$, " " " " " " " " " " " " superior "

$\lambda = \frac{\alpha'}{\alpha}$, " " " " equal "

$\alpha = 0$, " " " but one minimum at most and this at inf. conj.

$\alpha' = 0$, " " " " " " " " " " " " sup. "

$\alpha > 0$, " " " at least one minimum and this at superior conjunction.

$\alpha' > 0$, " " " " " " " " " " " " inferior "

$\alpha' = \alpha = 0$, " " " no minimum.

To illustrate these matters geometrically, we may choose an ellipse of eccentricity = 0.66 and of inclination $i = \cos^{-1}(0.25) = 75^\circ\,31'$ to represent the orbit of the system. For the larger body take a radius equal to $0.104a$ and for the smaller (and let us suppose also the darker) to $.0073a$, when a denotes the semi-major axis of the orbit. In the accompanying figures, the posi-

tion of the line of apsides AA lies with respect to the node line KK', as it would appear to an observer viewing it from a position in the perpendicular to the orbit plane.

Fig. 1

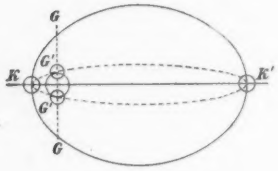


Fig. 2.

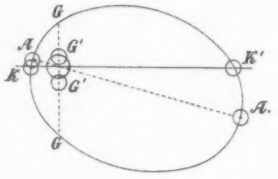


Fig. 3.

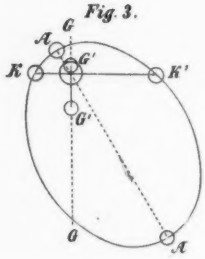


Fig. 4.

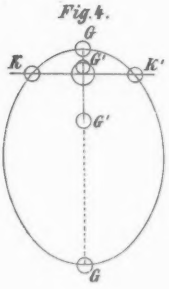


Fig. 6.

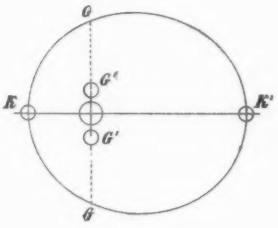


Fig. 5.

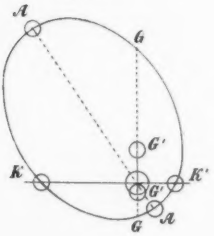
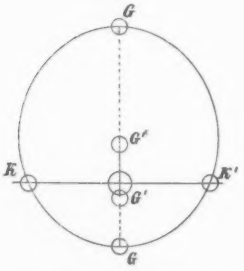


Fig. 7.



In all the figures the line of sight must be supposed to pass through the larger body and perpendicular to the plane of the drawing. In Fig. 1 the apparent orbit is represented by a dotted line and elsewhere only the positions GG, KK of the companion

at the principal epochs of light change are given. The minima, of course, occur when the positions GG of the secondary in its real orbit are such that its disc is projected wholly, or partially, before, or behind, the disc of the primary. The projections of GG are everywhere denoted by $G'G'$. In all the figures the G' above KK' represents the inferior conjunction of the satellite.

In Fig. 1 (Figs. here refer to the group just preceding) the line of apsides coincides with the node line, consequently, whatever the eccentricity, α must here equal α'. Hence the falling in brightness at these epochs will be equal or unequal according as $\lambda = \frac{\alpha'}{\alpha} = 1$, or $\lambda \lesseqgtr 1$. If the minima are unequal, the lower, or primary minimum falls at the inferior, or superior conjunction, according as $\lambda < \frac{\alpha'}{\alpha}$, or $\lambda > \frac{\alpha'}{\alpha}$. This follows directly from (1), since $L > 1$, or $L < 1$, according as $(1-\alpha)\lambda - (\lambda - \alpha') = \alpha\left(\frac{\alpha'}{\alpha} - \lambda\right)$ is positive, or negative. Since now α, $\frac{\alpha'}{\alpha}$ and λ are all essentially positive, this expression must be positive or negative according as $\lambda < \frac{\alpha'}{\alpha}$, or $\lambda > \frac{\alpha'}{\alpha}$. If $L > 1$ the primary minimum must, of course, fall at inferior conjunction and if $L > 1$, at superior conjunction.

For this position of periastron we should, of course, have the greatest possible difference of intervals between the minima.

On the hypothesis of the validity of Kepler's law in stellar systems, these intervals, as also those of the remaining figures, must be proportional to the areas $GK'G$ and GKG. The maxima obviously occur when both bodies present their full discs to the observer.

Fig. 2 exemplifies the condition of things after a motion of 15° of periastron has occurred. Here $\alpha' > \alpha$ and the minima will again be equal or unequal, according as $\lambda = \frac{\alpha'}{\alpha}$, or $\lambda \lesseqgtr \frac{\alpha'}{\alpha}$. But it is important to notice that if the minima were equal in the foregoing case, they must here be unequal and the reverse. At all events it is possible that an essential alteration in the star's mode of variation may have resulted from pure periastron change. In the event of unequal minima, whether the primary or secondary minimum falls at inferior conjunction, is determined by the same considerations as before.

Fig. 3 represents the case after a periastron motion of 60° has

ensued. Since α now equals 0, the minimum at superior conjunction has completely disappeared. The brightness now fluctuates between wider limits than before, since the disc of the companion during eclipse now encroaches deeper than formerly upon that of the central body. It is moreover to be noted that the star has now been transformed from a variable of two minima, into a variable with a single minimum. The sequence of changes from a variable with two equal minima into one with two unequal minima and finally into another with but one minimum, might be exhibited by the three stages. It is evident that if a variable loses one of its minima from this source, it must have done so by an *approach* of the secondary minimum *towards the middle of the period.*

In fig. 4, $\alpha = 0$ and α' is a maximum. The intervals Max. II — Min. and Min. — Max. I have become equal and between the brightness at the minima the greatest possible inequality obtains.

In fig. 5 the position of periastron has moved by 235°, α has become zero and the eclipse at inferior conjunction has disappeared, while the minimum at superior conjunction has reappeared.

Figs. 6 and 7 show what would happen if the eccentricity had decreased from 0.66 to 0.424 without a change in the position of the line of apsides.

In fig. 6, $\alpha = \alpha' = 0$ and the star no longer exhibits light changes, and this condition of things will prevail until the motion of the line of apsides brings a shorter radius vector between the point of conjunction and the central body as has occurred in fig. 7.

It is a note-worthy fact that in a system whose orbit has such a form and position as to give a period with two minima, the star can never completely lose its variable character through pure periastron motion. This might occur temporarily however with a period containing a single minimum, though the motion which removed the secondary minimum would in course of time restore it again. The point has been made sufficiently clear that the periastron motion, coupled with an alteration in either the inclination or the eccentricity may transform a variable whose light changes are explicable on the satellite theory, of either one or two minima into an invariable star.

Let us now derive the connection between α and α' and the orbital elements and confirm some of these conclusions algebraically.

Without restricting the generality of the problem we may put

$$\rho' < \rho.$$

We have also $\rho' = r' \cos \sigma$ and $\rho = r \cos \sigma$.

From the laws of elliptical motion,

$$r = \frac{p}{1 + e \cos \tilde{\omega}} \qquad \text{and} \qquad r' = \frac{p}{1 + e \cos (\pi - \tilde{\omega})},$$

whence

$$\frac{\rho'}{\rho} = \frac{r'}{r} = \frac{1 - e \cos \tilde{\omega}}{1 + e \cos \tilde{\omega}}. \tag{2}$$

From the oblique triangles GOM and EOM of Fig. 1, we find

$$\alpha = x - \frac{1}{2} \sin 2x + \varkappa^2 y - \frac{1}{2} \varkappa^2 \sin 2y$$

$$\alpha' = x' - \frac{1}{2} \sin 2x' + \varkappa^2 y' - \frac{1}{2} \varkappa^2 \sin 2y'.$$

Writing for brevity

$$\delta = \cos x = \frac{(1 - \varkappa^2)(1 - e \cos \tilde{\omega})^2 + \rho'^2 (1 + e \cos \omega)^2}{2\rho' (1 - e^2 \cos^2 \tilde{\omega})} \tag{3}$$

and

$$\delta' = \cos y = \frac{(\varkappa^2 - 1)(1 - e \cos \tilde{\omega})^2 + \rho'^2 (1 + e \cos \tilde{\omega})^2}{2\varkappa\rho' (1 - e^2 \cos^2 \tilde{\omega})} \tag{4}$$

$$\Delta = \cos x' = \frac{1 - \varkappa^2 + \rho'^2}{2\rho'} \tag{5}$$

$$\Delta' = \cos y' = \frac{-1 + \varkappa^2 + \rho'^2}{2\varkappa\rho'}. \tag{6}$$

All these are readily written from the triangles mentioned above.

We may now write

$$(7) \begin{cases} \alpha = \cos^{-1} \delta - \frac{1}{2} \sin 2 [\cos^{-1} \delta] \\ \qquad + \varkappa^2 \cos^{-1} \delta' - \frac{1}{2} \varkappa^2 \sin 2 [\cos^{-1} \delta'] \\ \alpha' = \cos^{-1} \Delta - \frac{1}{2} \sin 2 [\cos^{-1} \Delta] \\ \qquad + \varkappa^2 \cos^{-1} \Delta' - \frac{1}{2} \varkappa^2 \sin 2 [\cos^{-1} \Delta']. \end{cases}$$

From (7) and the definitions of δ, δ', Δ and Δ' it is seen, that $\frac{\alpha'}{\alpha}$, α' and α depend only upon $\varkappa$ and the elements of the orbit, since

$$\rho' = \frac{p \cos \sigma}{1 + e \cos \tilde{\omega}}, \text{ and } \sigma \text{ is given by } \sin \sigma = \sin i \sin \tilde{\omega}. \tag{8}$$

Inasmuch as the character of the star's variability depends wholly on $\frac{\alpha'}{\alpha}$, α' and α, it is clear that the orbital elements together with $\varkappa$, determine it.

From (3)—(6) it is apparent that α varies continuously with ρ', between $(1-\varkappa)\dfrac{1-e\cos\tilde{\omega}}{1+e\cos\tilde{\omega}}$ and $(1+\varkappa)\dfrac{1-e\cos\tilde{\omega}}{1+e\cos\tilde{\omega}}$.

α approaches 1, or 0, according as ρ' approaches the first or second of these values. α' is a continuous function of ρ' between $1-\varkappa$ and $1+\varkappa$, when ρ' approaches $1-\varkappa$, α approaches 1, and when $\rho'=1+\varkappa$, $\alpha'=0$. If the form and position of the orbit are such that,

$$\frac{1-\varkappa}{\cos\sigma}\frac{1-e\cos\tilde{\omega}}{1+e\cos\tilde{\omega}} < r' < \frac{1+\varkappa}{\cos\sigma}\frac{1-e\cos\tilde{\omega}}{1+e\cos\tilde{\omega}},$$

$$\text{or more simply } \frac{1-\varkappa}{\cos\sigma} < r < \frac{1+\varkappa}{\cos\sigma},$$

it follows that

$$(9)\qquad (1-\varkappa)\frac{1-e\cos\tilde{\omega}}{1+e\cos\tilde{\omega}} < \rho' < (1+\varkappa)\frac{1-e\cos\tilde{\omega}}{1+e\cos\tilde{\omega}}$$

and consequently, neither α nor α' equals zero, and the variable must have two equal or unequal minima, according as

$$\lambda = \frac{\alpha'}{\alpha}, \text{ or } \lambda \lessgtr \frac{\alpha'}{\alpha}.$$

If $\rho' > (1+\varkappa)\dfrac{1-e\cos\tilde{\omega}}{1+e\cos\tilde{\omega}}$, α equals 1 and we have at least one minimum. But if

$\rho' > (1+\varkappa)\dfrac{1-e\cos\tilde{\omega}}{1+e\cos\tilde{\omega}}$, we shall have one minimum at most.

Similar conclusions follow for α', if ρ' lies without the limits $1-\varkappa$ and $1+\varkappa$. So soon as $\rho > 1+\varkappa$, the star ceases to be a variable.

From (8) it appears that ρ' satisfies (9), if $\cos\tilde{\omega}$ satisfies

$$(10)\qquad \frac{1}{e}-\frac{p}{e}\frac{\cos\sigma}{1-\varkappa} < \cos\tilde{\omega} < \frac{1}{e}-\frac{p\cos\sigma}{e(1+\varkappa)}.$$

From this, $\rho < 1+\varkappa$, if

$$\cos\tilde{\omega} > \frac{p\cos\sigma}{e(1+\varkappa)}-\frac{1}{e}.$$

If now

$$(11)\qquad -\cos\tilde{\omega} > \frac{p\cos\sigma}{e(1+\varkappa)}-\frac{1}{e} < \cos\tilde{\omega},$$

we have,

$$(1+\varkappa)\frac{1-e\cos\tilde{\omega}}{1+e\cos\tilde{\omega}} < \rho' < 1+\varkappa.$$

But if

$$\cos\tilde{\omega} > \frac{p\cos\sigma}{e(1+\varkappa)}-\frac{1}{e}, \text{ we obtain } \rho' > 1+\varkappa.$$

Recapitulating then, we may say, that if the orbit of the star

has such a position as to satisfy (10), the light curve must exhibit two minima, which may be either equal or unequal. If now, periastron should shift its position until (11) is satisfied one minimum would disappear, and if the motion should continue until (12) becomes true, the star must cease temporarily to show light fluctuations, since in this case $a = a' = 0$. A continuation of the motion however, might bring cos $\tilde{\omega}$ between its former limits, whereupon the star must again resume its light changes, feebly at first, but with ever rising emphasis, until in due course, a maximum value of the amplitude of light variation shall be reached, after which the process will be repeated.

Since cos $\tilde{\omega}$ may assume values between its limits for which $\lambda = \frac{a'}{a}$, *i. e.* for which the minima become equal, we may readily conceive how these simple and familiar periodic changes of the orbital elements make it possible for a "double variable" to show a period of light change with two equal or unequal minima at one epoch and with a single minimum at another. It is also just as easily conceivable that a star may lose its variable character entirely for a time and resume it again or not at a later epoch according as the inclination or eccentricity remain constant, or undergo change in the mean time. It is even possible that the foregoing elementary considerations may account for the unexpected appearance of variable stars in places, where, previously, or subsequently, invariable stars have been recorded. They may also contribute to the explanation of the enigmatical flutterings of brightness of some temporary stars.

UNIVERSITY OF ILLINOIS, Urbana, Ill.,
Jan. 26th, 1898.

VARIABLE STAR CLUSTERS.

EDWARD C. PICKERING.

Since the announcement made in Circular Nos. 2 and 18, of variables discovered in clusters, a further examination of the clusters ω Centauri, Messier 3, Messier 5, and N. G. C. 7078 has been made by Professor Bailey. As a result, the numbers of known variables in these clusters have been increased by 62, 19, 22, and 24 respectively, making the total numbers 122, 132, 85, and 51, or 390 in all four clusters. Adding to these the 47 already announced in other clusters, makes the total number 437.

New Variable Stars.

When a new variable star is discovered at this Observatory it is the custom to collect all the photographs of the region containing it and to derive its photographic magnitude from each of them as described in the H. C. O. Annals, Volume XXVI, page 250. We can thus determine its brightness on from twenty to a hundred or more nights distributed over the last ten years. The approximate dates of maxima, the corresponding magnitudes, the period and the form of light curve are also determined so far as possible. Examples of such results have already been published, but every year, owing to the increasing amount of material, the work becomes more laborious although at the same time more complete and exact. Many of these stars vary irregularly so that their elements cannot be determined precisely. When the object is not a catalogue star its position, and that of each of the fainter comparison stars must also be determined from measures of their rectangular coördinates. An attempt is then made to photograph each of these variables once a month, and, if possible, to obtain corresponding observations of their visual magnitudes. As the total number of variable stars discovered here is now more than a hundred, not including those found in clusters, the labor involved in this work is very great. Accordingly, it is difficult to reduce all the required data for one star before another is found. The accompanying table gives the material that has been so far collected for the variables recently discovered here from the Draper Memorial photographs:

Constellation.	Designation.	R. A. 1900.	Decl. 1900.	Type.	No. Plates.	Magn. Br	Magn. Ft.	Discoverer.
		h. m.	° ′					
Eridanus	— 16° 771	3 59.8	— 16 0	III	35	8.3	9.4	M. Fleming.
Eridanus	— 25° 1766	4 7.3	— 25 24	III	65	8.1	< 12.5	M. Fleming.
Monoceros	— 8° 1641	6 52.5	— 8 56	III	43	8.1	10.3	M. Fleming.
Puppis	— 38° 4049	8 1.7	— 38 29	IV	...	..	...	L. D. Wells.
Puppis	— 22° .160	8 3.1	— 22 38	IV	...	..	...	L. D. Wells.
Hydra	— 5° 2550	8 24.7	— 5 59	III	...	..	...	M. Fleming.
Carina	R	10 40.9	— 58 54	...	149	9.6	10.7	L. D. Wells.
Virgo	— 5° 3424	12 2.1	— 6 12	III	...	..	...	M. Fleming.
Centaurus	.....	13 15.1	— 61 3	III	...	..	...	M. Fleming.
Apus	A.G.C. 19014	13 55.6	— 76 19	III	...	..	...	—
Bootes	+ 14° 2700	14 1.7	+ 13 59	III?	...	..	...	L. D. Wells.
Libra	— 17° 4122	14 30.3	— 17 36	II?	38	8.3	9.6	E. F. Leland.
Triang. Aust.	A.G.C. 20554	15 4.8	— 69 42	IV	85	9.1	9.8	—
Serpens	+ 10° 2956	16 2.5	+ 10 12	III	41	9.0	< 11.9	M. Fleming.
Ara	A.G.C. 23005	16 54.3	— 54 55	IV	...	..	...	L. D. Wells.
Pavo	A.G.C. 23935	17 34.7	— 57 40	IV	...	..	...	L. D. Wells.
Pavo	.....	17 41.1	— 62 23	III	65	9.1	< 12.8	M. Fleming.
Ara	.....	17 45.7	— 51 40	III	...	..	...	M. Fleming.
Cygnus	+ 32° 3522	19 37.1	+ 32 23	IV	...	..	...	L. D. Wells.
Pavo	A.G.C. 27560	20 3.3	— 60 14	III	...	..	...	M. Fleming.
Capricornus	A.G.C. 27776	20 11.3	— 21 38	IV	55	8.6	10.3	—
Micros'opium	— 40° 13888	20 22.6	— 40 45	III	70	8.5	< 12.5	M. Fleming.
Capricornus	— 17° 6181	21 1.7	— 16 49	III	79	8.1	9.3	M. Fleming.
Aquarius	— 14° 5960	21 7.3	— 14 48	III	78	8.4	9.3	M. Fleming.
Indus	A.G.C. 29232	21 13.6	— 45 27	IV	...	..	...	L. D. Wells.
Andromeda	+ 47° 4318	23 50.3	+ 48 5	III	48	9.3	9.8	M. Fleming.
Cassiopeia	.....	23 58.2	+ 55 7	III	101	9.8	< 13.4	M. Fleming.

R. A. $3^h\ 59^m.8$. Bright hydrogen lines suspected.

R. A. $4^h\ 7^m.3$. Hydrogen lines bright.

R. A. $6^h\ 52^m.5$. Hydrogen lines bright.

R. A. $8^h\ 1^m.7$. Found to be fourth type by Mrs. Fleming.

R. A. $8^h\ 24^m.7$. Bright hydrogen lines suspected.

R. A. $10^h\ 40^m.9$. This star is No. 119 on page 627 of the Argentine General Catalogue.

R. A. $12^h\ 2^m.1$. Bright hydrogen lines suspected.

R. A. $13^h\ 15^m.1$. Hydrogen lines bright. The position of this star for 1875 is R. A. $= 13^h\ 13^m\ 31^s$, Decl. $= -60°\ 55'$.

R. A. $13^h\ 55^m.6$. Bright hydrogen lines suspected. In the Uranometria Argentina, page 243, this star which is θ Apodis, is stated to be variable. Discovered independently by Mrs. Fleming by means of its spectrum. The photographs show a variation of about one magnitude.

R. A. $15^h\ 4^m.8$ In Argentine General Catalogue "var?" Discovered independently from photographic charts by Miss L. D. Wells.

R. A. $16^h\ 2^m.5$ Hydrogen lines bright.

R. A. $17^h\ 41^m.1$. Hydrogen lines bright. The position of this star for 1875 is R. A. $= 17^h\ 38^m\ 45^h$, Dec. $= -62°\ 21'.6$.

R. A. $17^h\ 45^m.7$. Hydrogen lines bright. The position of this star for 1875 is R. A. $= 17^h\ 43^m\ 42^s$, Dec. $= -51°\ 31'.2$.

R. A. $20^h\ 3^m.3$ Bright hydrogen lines suspected.

R. A. $20^h\ 11^m.3$. Suspected of variability by Secchi and others. Found independently from the photographs by Miss L. D. Wells.

R. A. $20^h\ 22^m.6$. Hydrogen lines bright. Maxima represented by formula, $2410860 + 325$ E.

R. A. $23^h\ 50^m.3$. Bright hydrogen lines suspected.

R. A. $23^h\ 58^m.2$. Hydrogen lines bright. The position of this star for 1855 is R. A. $= 23^h\ 55^m\ 53^s$, Decl. $= +54°\ 52'.3$.

In Circular No. 10 the variability of $-27°$ 15202 (erroneously printed 15203) suspected by Thome and was confirmed by Miss E. F. Leland. Measures of 35 photographs give the maximum brightness 8.9, minimum <12.3.

In Circular No. 17 the variability of a star in R. A. $= 0^h\ 25^m.5$, Dec. $= -46°$ 58′ (1900) is announced. Measures of 26 photographs give the maximum brightness 9.0, minimum <12.2.

In Circular No. 17 the variability of a star in R. A. $= 13^h\ 31^m.1$, Dec. $= -55°$ 58′ (1900) is announced. Measures of 42 photographs give the maximum brightness 9.0, minimum <12.6.

In Circular No. 17 the variability of a star in R. A. $= 20^h\ 8^m.5$ Dec. $= -44°$ 43′ (1900) is announced. Measures of 114 plates give the maximum brightness 9.0, minimum <11.4.

In Circular No. 19 the variability of a star in R. A. $= 5^h\ 18^m.9$, Dec $= -69°$ 21′ (1900) is announced. Measures of 51 photographs give the maximum brightness 8.2, minimum 9.4.

January 31, 1898,

Circular No. 24. H. C. Observatory, Cambridge, Mass.

LIST NO. 6 OF NEBULÆ DISCOVERED AT THE LOWE OBSERVATORY ECHO MOUNTAIN, CAL., FOR 1900.

LEWIS SWIFT.

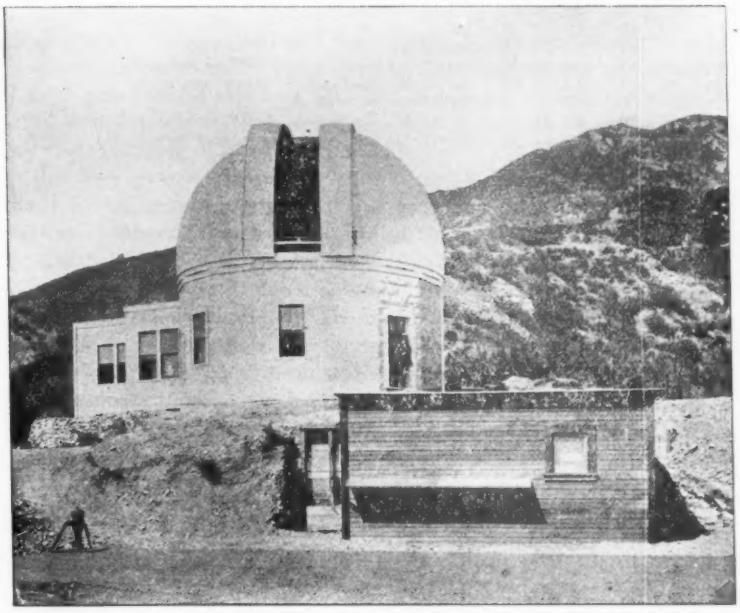
FOR POPULAR ASTRONOMY.

No.	Date.	R. A. h m s	Decl. ° ′ ″	Description.
1	Dec. 25	2 11 10	−31 39 15	*pF. pS. R.* distant *D*nf.*
2	" 1	2 25 45	−34 42 0	*eeeF. S. R. D** nearly *p. np* of 2.
3	" 22	2 26 0	−34 41 42	*eF. eS. R. F*nrn. D*np. Sf* of 2.
4	" 22	2 26 15	−36 28 55	*pB. pS. vE.*
5	" 22	2 37 0	−28 37 0	*eeF. S. R.* 3 fine *Dstnf.* each about 7″.
6	" 22	2 40 35	−28 22 35	*eeF. S. R. D*np.*
7	" 22	3 7 30	−25 42 0	*eeF. S. R. 2fst* near *Sp* point to it.
8	" 26	3 35 40	−27 11 10	*eE. eF. lE.* * close *nf.* stellar.
9	" 26	3 50 40	−28 26 15	*eF. S. F** in contact *nf.*
10	" 23	4 8 30	−32 49 48	*vF. vS. R.* resolvability suspected. 1531-2 in field.
11	" 26	4 52 0	−28 41 35	*eeeF. pL. iR. D** 24 *Sf* point to it.
12	Nov. 30	5 2 30	−20 37 15	*eeeF. pS. bet* 2*st.* close to *eeF D* eedif.*
13	Dec. 26	5 15 10	−25 11 30	*eeeF. vS. R.* 7^{m}* 15^{s} *p* little *S* nearly obliterates it.
14	" 1	5 30 0	−23 36 30	*eeF. pS. R.* 7^{m}* close *f. Sf* of 1880.
15	Nov. 30	5 44 0	−30 31 57	*eeeF. pS. R, F* nrp. sev Bst Sf.* 3*stn* curved.
16	Dec. 1	5 53 25	−23 11 30	*pB. pS. R* in vacancy. *sev BstSf.*
17	" 1	5 56 45	−23 41 30	*B. L. R. bet* * *nf* and wide *D* np.*
18	" 26	6 1 5	−27 51 50	*vF. pS. lE.* * in contact *np. nend* like a brush.
19	" 28	9 18 10	−32 2 55	*pF. cS. vE.* 10^{m}* close *Sp.*
20	" 28	9 54 5	−26 42 35	*eeeF. eeS. R. eFD** close *S. eedif.* 3078 in field.
21	" 28	10 11 20	−33 03 10	*eeF. pS. iR.* in center of trapezium.
22	" 30	10 16 38	−33 46 15	*vF. cS. R.* 9^{m}* pretty close *f.*
23	" 30	10 24 30	−35 3 15	*eeeF. eeS. R eF** in contact *S. f* of 2. Note.
24	" 30	10 35 12	−35 31 35	*eeeF. eeS R. eF** in contact *S. f* of 2. Note.
25	" 29	10 36 0	−35 5 25	*eeeF. eS. R. eeFD** near *S.*

Notes to numbers 23 and 24. The descriptions of these two nebulæ as will be seen are identical in every particular. I observed a very faint nebula which on examination of N. G. C. proved to be No. 3267. Near following was an exceedingly faint double star, which with a power of 132, looked as if the north one was an exceedingly small nebula of unimagined faintness. With 200, my suspicion that it was not a star was confirmed. Soon I ran across another presenting the same appearance, both being north of the star, and at the same distance (about 4″) and of the same size and brightness or rather faintness. These minute objects if as distant as their companion stars must in volume vastly exceed the orbit of Neptune and yet are self luminous.

ECHO MOUNTAIN, California.

LOWE OBSERVATORY, ECHO MOUNTAIN, CALIFORNIA.



LIST NO. 7 OF NEBULÆ DISCOVERED AT THE LOWE OBSERVATORY.

LEWIS SWIFT.

FOR POPULAR ASTRONOMY.

No.	Date.	R. A 1900.0 h	m	s	Decl. 1900.0 °	′	″	Description.
1	Jan. 22	9	23	0	−42	24	30	*pB. S. R.* 7^{m*} *nf. D***p*
2	Dec. 30	9	45	0	−32	23	5	*pB. pS. eeE.* spindle 7^{m*} *np. np.* of 2. Not 3038.
3	" 30	9	45	35	−32	25	5	*eeF. eS.* 3 *Fst* close *f. sf* of 2.
4	" 30	9	52	32	−31	48	0	*vF. S. R.* 7½^{m*} *np.* 2 or 3 *Fst* near.
5	" 30	10	24	30	−35	3	15	*eeeF. eeS. R eF** in *con sp* of 2.
6	Jan. 1	10	26	3	−28	12	30	*pF. vS. R.* trapezium *nr sp.*
7	" 1	10	27	10	−29	52	35	*eeeF pL R D** *nr sf.** with *dis com f* and *p.*
8	Dec. 30	10	35	12	−35	31	35	*eeeF. eeS. R. eF** in con *st* of 2.
9	Jan. 1	11	17	0	−28	27	30	*pB. pS. R.* 10^{m*} close *n lf.* 7^{m*} *t.*
10	Dec. 29	11	49	35	−37	21	5	*eF. vS.* 7^{m*} *Sp.*
11	" 30	12	0	30	−27	22	45	*vF. L. lE.* 8^{m*} *nrf. np* of 2.
12	Jan. 1	12	0	35	−27	24	25	*eeeF. pL. eE.* 3 8^{m} stars *f. sf* of 2.
13	" 31	12	3	25	−31	2	15	*pB. vS.* * close *sf. ve.* 45°.
14	" 1	12	14	28	−25	37	15	*pB. S. R. bet* 4*st sf* and 8^{m*} *np.*
15	" 30	12	18	45	−39	14	0	*pF. vS. R.* close *p* of *N. G. C.* 4373. Note.
16	" 1	12	20	0	−25	30	15	*eF. vS. R. bet.* 7^{m*} *st* and 8^{m*} *np. vdif.*

No.	Date.	R. A. 1900.0 h m s	Decl. 1900.0 ° ′ ″	Description.
17	" 30	12 22 5	— 38 48 50	*pB. pL. R.* 7^{m}* with *dis com nrp.*
18	" 1	12 45 0	— 25 22 15	*eeeF. S. R.* 8^{m}* *nf.*
19	" 31	12 46 0	— 29 17 35	*B. S. lE.*
20	" 31	13 11 10	— 31 33 45	*pB. pS. R.* 9^{m}* *nr sf.*
21	" 31	13 12 5	— 31 7 45	*eeeF. pL. R.* 9^{m}* *nr sp. edif.*
22	" 31	13 18 25	— 29 47 37	*eeF. pS. R.* trapezium *nr sf.*
23	" 30	13 31 0	— 33 35 0	*pF. pS. R.*
24	" 30	13 31 10	— 33 33 55	*eeeF. eeS.* like *D**, one *com* nebulous. Note.
25	" 30	13 51 35	— 39 31 50	8^{m}* in center of *eeeF* nebulosity. Note.

NOTES.

A few of the above are very interesting objects, some for their size and brightness, and remaining so long undiscovered: others for their extreme faintness.

No. 15. This is a double nebula, its companion being G. C. 2928 = N. G. C. 4373. As it is close to the above and nearly as bright though much smaller, it seems strange that it should have been missed.

No. 24. This is another and the 4th I have found resembling a close *D* star. All have been about 4″ apart. They are doubtless physically connected, one being a star the other a stellar nebula.

No. 25 is a nebulous star, the only one I have ever discovered. The central star is of the 8th magnitude, environed in an exceedingly faint luminous atmosphere. An 8th magnitude star follows 15^{s}, which was free from nebulosity.

ECHO MOUNTAIN, California.

THE MOTION OF A HEAVENLY BODY IN A RESISTING MEDIUM.

GILBERT AMES BLISS.

FOR POPULAR ASTRONOMY.

The problem of motion in a resisting medium is to find the effect of such a resistance upon the motion of a planet or comet, revolving around the Sun.* It is needless to say that the planets have never shown any irregularities of movement which could be attributed to this cause; and such a state of affairs agrees exactly with theory, because the densities of the planets are enormously great as compared with the supposed density of the ether. The motion of a planet in ether is like that of a bullet falling in air; the effect of the resistance is very slight, and a long period of time is necessary before the planet can be retarded to a degree at all

* See Tisserand's *Mecanique Celeste* Vol. IV., Ch. XIII.

sensible. But with comets the conditions are different. Their densities are extremely small as compared with those of the planets, and in absolute value are very much less than the best vacuum obtainable with an air-pump. It would therefore seem possible, or at least plausible, that the resistance of the ether might affect a comets' motion; and as a matter of fact one comet (Encke's) has shown irregularities which can be explained only by the existence of a resisting medium.

But before definite investigations can be undertaken, it is necessary to find the law of resistance to motion in a medium. The opposing force depends upon three things, (1) the cross section of the body, (2) its velocity, and (3) the density of the medium. The effect of the first is evident, for the larger the body the harder it would be to move it in any substance. But the law as dependent upon velocity is not so apparent. It has been found that the resistance in air increases with the velocity, but not in proportion to the first power. Some very good authorities consider that it may be expressed by a series involving the velocity to the fourth power at least. Thus.

$$R = a + bV + cV^2 + dV^3 + eV^4$$

where a, b, c, etc., are constants. But no law has ever been deduced from theoretical considerations. Similarly it is easy to see that the resisting force is proportional to the density of the medium.

As a well-established law of resistance is lacking, one has been assumed which is very general; namely,

$$(1) \qquad R = \frac{\varkappa S}{m} \rho V^p = h \rho V^p$$

where R is the resistance per unit of mass, S is the cross-section, m the mass, V the velocity of the body, and ρ is the density of the medium. In the case of the solar system where the Sun is the powerful center of attraction, it is probable that the central ether layers are more dense than those far away, so that the density at any point in space may be expressed as inversely proportional to r, the distance of the point from the Sun. Thus

$$\rho = \frac{1}{r^q}$$

The equation (1) becomes;—

$$(2) \qquad R = \frac{\varkappa S}{m} \frac{V^p}{r^q} = h \frac{V^p}{r^q}$$

Expressed in words, this means that the resistance is directly proportional to cross-section and to some power of the velocity (p being unknown), and inversely proportional to some power of the distance from the Sun. The generality of the assumption lies in the fact that p and q are entirely arbitrary. It is very curious that whatever the values of p and q, the results obtained are almost the same, and the larger p and q are taken the nearer these results are to coincidence.

The law of resistance being thus assumed, it remains to describe its effect upon the orbits of the planets and comets. To one unacquainted with the problem it might seem possible to derive the curve into which the orbit is transformed under the influence of the resisting force. But such a direct solution is impossible because differential equations are involved which can not be integrated by known methods. There is, however, another method which enables us to obtain readily very approximate results. It is the calculation of the variations in the elements of elliptic motion, caused by the resistance. The elements which determine the orbit in which the body moves, are six in number. Two fix the position of the orbit plane in space with respect to the plane of the ecliptic. They are the inclination denoted by i, and the longitude of the ascending node denoted by Ω. The inclination is the angle between the two planes, and Ω is the angular distance in the plane of the ecliptic between x-axis and the ascending node. Two other elements, the major-axis denoted by a, and the eccentricity denoted by e, determine the size and shape of the ellipse itself. And finally, the position of the ellipse in the orbit plane is defined by ω, the longitude of perihelion, and ε the longitude of the mean epoch. ω is the sum of two angles, Ω in the plane of the ecliptic, and the angle between the line of nodes and perihelion in the orbit plane. Similarly ε is the sum of Ω, and the angle between the line of nodes and the mean epoch. The epoch T is usually given as the sixth element, but in the present instance it is more convenient to use an angle such as ε.

If these six elements of an elliptical orbit are given, the orbit itself can be determined in space. To solve the problem of a resisting medium the orbit is assumed to be always an ellipse. The elements of elliptic motion are known at present, and the alterations which will be produced in them by the resistance can be calculated. Consequently the shape of the orbit at any future time, and its position in space can be deduced.

As was said above the problem is to find the variations in the elements caused by the resistance. These variations are of two

kinds, periodic and secular. An element varies periodically when it first increases and then decreases by the same amount, or vice versa, in a definite period. Such variations are unimportant because they are small and have periods lasting not more than one revolution of the body. But the secular variations are important. They accumulate from year to year and if given time enough produce very marked changes in the orbit.

The general equations expressing the variations of the elements are as follows:

$$(3)\quad \begin{cases} \dfrac{di}{dt} = 0 \\ \dfrac{d\varOmega}{dt} = 0 \\ \dfrac{d\omega}{dt} = -\dfrac{2R\sqrt{1-e^2}}{nae}\dfrac{\sin v}{\sqrt{1+e^2+2e\cos v}} \\ \dfrac{d\varepsilon}{dt} = \dfrac{2Re}{na}\dfrac{\sin v}{\sqrt{1+e^2+2e\cos v}}\left\{\dfrac{r}{a} - \dfrac{\sqrt{1-e^2}}{1+\sqrt{1-e^2}}\right\} \\ \dfrac{da}{dt} = -\dfrac{2R}{n\sqrt{1-e^2}}\sqrt{1+e^2+2e\cos v} \\ \dfrac{de}{dt} = -\dfrac{2R\sqrt{1-e^2}}{na}\dfrac{\cos v + e}{\sqrt{1+e^2+2e\cos v}} \end{cases}$$

where n is the mean angular velocity of the body; and v is the true anomaly, or the angular distance which the body has moved from perihelion. It will be remembered that e, the eccentricity of an ellipse, is always less than unity and greater than zero; and a, n, and R, are positive quantities; so that the only factors in these equations which can change sign are $\sin v$ and $\cos v$. This is important because it is known by calculus that the elements increase when the right member is positive, and decrease when it is negative.

For the benefit of those interested in the mathematical side of the problem, the present paragraph indicates briefly how the six equations above are derived. The problem of two bodies is expressed by three equations:

$$\frac{d^2x}{dt^2} + \frac{k^2x}{r^3} = 0$$

$$\frac{d^2y}{dt^2} + \frac{k^2y}{r^3} = 0$$

$$\frac{d^2z}{dt^2} + \frac{k^2z}{r^3} = 0$$

They can be completely solved, and their solution defines the orbit in terms of the six elements. Now suppose a perturbing force introduced, such as a resistance. Then these equations become:

$$\frac{d^2x}{dt^2} + \frac{k^2x}{r^3} = \frac{dR}{dx}$$
$$\frac{d^2y}{dt^2} + \frac{k^2y}{r^3} = \frac{dR}{dy}$$
$$\frac{d^2z}{dt^2} + \frac{k^2z}{r^3} = \frac{dR}{dz}$$

where $\frac{dR}{dx}$, $\frac{dR}{dy}$, and $\frac{dR}{dz}$, represent the projections of the perturbing force on the x, y and z axes respectively. The last three equations in the present instance and in general, can not be completely solved. But by considering the constants found in the simple problem of two bodies not as constants but as functions of the time, Lagrange has shown the effect of the perturbing force upon them. Jacobi has deduced the same results, but in a different manner. The six equations given above are derived by Jacobi's method for the particular case of a resisting medium. This method of solution in general, is called the method of *variation of constants.**

Consider first the variations of the elements i and Ω, which define the position of the orbit plane in space. The resisting force is always directly opposed to motion; that is, it acts along the tangent to the ellipse at the point where the body happens to be, and in the direction opposite to that of the body's movement. The tangent and the force directed along it are always in the plane of motion, and for this reason the position of the plane cannot be altered. i and Ω are therefore constant as indicated by their equations, and the orbit plane remains stationary in space.

It will be observed that the element increases when the right member of the equation expressing its variation is positive, and decreases when it is negative. Since e is less than unity, the expression for $\frac{d\omega}{dt}$ is negative when v varies from 0 to π in the first half-revolution, and positive in the second half where v varies from π to 2π; and to each negative value corresponds a positive value of the same magnitude. The longitude of perihelion there-

* See Tisserand's Mécanique Céleste, Vol. I, Ch VII and IX.
NOTE —We have not yet received our partial differential signs.—ED.

fore, first decreases, and then increases by exactly the same amount. In the same way it may be shown that ε, the longitude of the mean epoch, has a periodic variation with a period of one revolution. Interpreted geometrically these statements mean that the direction of the major-axis, and the mean epoch, oscillate slightly about their average positions in the orbit plane, the period of oscillation being one revolution.

The variations of the four elements so far considered have been either zero or periodic. But those of a and e, the major-axis and eccentricity, are of the second kind, that is, secular. For the major-axis the deduction is simple. $\frac{da}{dt}$ is always negative, and on account of the term involving cos v, it repeats the same values in successive revolutions. Hence a constantly diminishes in length and, strange to say, by the same amount in each revolution.

Two other quantities, which are dependent upon a, are affected by its variation; namely, the mean angular velocity n, and the period P. From the theory of elliptic motion they are expressed in terms of a by the two equations:

$$n = \frac{k}{a^{\frac{3}{2}}}$$

$$P = \frac{2\pi a^{\frac{3}{2}}}{k}$$

where k is a constant. As a decreases the mean angular velocity increases, and the period decreases. In other words the resistance has the curious effect of shortening the time of revolution. Similarly the latus rectum, which depends upon both a and e, diminishes. It is expressed as follows:

$$p = a\,(1 - e^2)$$

But the diminution is not in proportion to that of a. If it were, the successive orbits would be similar ellipses, each one a little smaller than the preceding. The discussion of e shows that this is not the case.

The expression for $\frac{de}{dt}$ involves the same quantities which appear in that for the major-axis, but there is also in the numerator a factor $(\cos v + e)$ which is positive or negative according to the value of v. The eccentricity, therefore, sometimes increases and sometimes diminishes, so that it is impossible to tell without further consideration whether on the whole e is augmented or the

opposite. The same equation developed farther gives place to the following:

$$\delta e = -2h' \left(A_0 e + \frac{1}{2} A_1\right) nt$$

where δe represents the variation of e; h' is a constant, and t is the time during which e varies. A_0 and A_1 represent integrals which are positive when $p > 1$ and $q > 2$, p and q being the same which appear in the formula for the resistance. It is certain that the resistance must depend upon higher powers of V and r than the first and second, so that it is safe to say that the eccentricity always decreases. The eccentricity for an ellipse is less than unity, and as it approaches zero the ellipse becomes more and more nearly a circle. It is at once apparent that resistance tends to transform an elliptical orbit into a circular one.

All these results may be summed in a few words. (1) The resisting medium does not affect the position of the orbit plane; (2) the direction of the major-axis, and the mean epoch oscillate slightly during each revolution; and (3) the major-axis and eccentricity decrease continually. Suppose at any instant the resisting medium removed, and the body moving undisturbed in an elliptical orbit. Then the discussion shows that if the same conditions be repeated at any succeeding instant, the body will be moving in an ellipse more nearly circular, with a shorter major-axis and period. The actual path through the resisting medium is a spiral, at first elliptical, then becoming more nearly circular and centering at the Sun. The angular motion of the body is like that of a stone at the end of a string which winds about a stick held in the hand. During the first few revolutions while the string is long, the stone moves slowly; but as the string winds up, the stone approaches the stick and its motion becomes very rapid.

So far the results have been purely qualitative, not quantitative. The inclination and longitude of the node do not vary, and the longitudes of perihelion and of the mean epoch only periodically, so that the amounts of the variations of these four are of no interest. The calculation of the variation of a and e is comparatively simple in the case of the planets, where the eccentricity is small and its second power may be neglected. By developing the right members of the equation for $\frac{da}{dt}$ and $\frac{de}{dt}$ into Fourier series, integrating, and neglecting e^2, two very compact expressions are found, namely

$$\frac{\delta a}{a} = -2h'nt$$

$$\delta e = -h'nt\,(p+q-1)\,e$$

where δa and δe represent the variations in the elements, and t is the time during which the variations were developed, h' is defined by the following equation:

$$h' = \varkappa \frac{S}{m} \frac{k^{p-2}}{P^{\frac{p}{2}+q-2}}$$

P is the latus rectum. From these two relations the perturbations of the planets can be calculated when the density of the resisting medium is known. m and P are very large as compared with S and k so that in each case the variation must be very small, unless the period of time is very great.

For the comets it is found convenient to use two other quantities, n the mean angular velocity and an angle φ, instead of a and e. φ is defined by the equation:

$$e = \sin \varphi$$

By a similar process of developement and integration equations for these quantities are found as follows:

$$\frac{\delta n}{n} = \frac{3h'}{\cos^2\varphi}\,[A_0\,(1+e^2) + A_1 e]\,nt$$

$$\delta\varphi = -\frac{h'}{\cos\varphi}\,[2A_0 e + A_1]\,nt$$

or by combining these two

$$\frac{\delta n}{n} = -\frac{3\delta\varphi}{\cos\varphi} H_{pq}$$

$$H_{pq} = -\frac{\cos\varphi\,\delta n}{3n\delta\varphi}$$

where H_{pq} is evidently a function of A_0 and A_1. When p and q are given H_{pq} can be determined, and curiously enough it is found to be practically a constant, with a value between 0.9 and 1.0. The right member of this equation should be a constant for any body, planet or comet.

Encke's comet showed irregularities in movement like those above described, but with the remarkable difference that the variations themselves decreased. In the period of years 1819-1865, n the angular velocity, and the eccentricity varied appreciably. The function H_{pq} was found to have a value 0.97. But in the periods 1865-1871 and 1871-1881, the variations were propor-

tionately much less. Tisserand has suggested an explanation in the fact that the comet itself changed in size. Another theory is that the comet was retarded not by the influence of the ether, but by meteoric streams. The latter seems the more probable because it allows an explanation of the fact that so far as is known no other comets have been retarded in a similar manner. If this theory is the true one then the orbits of the other comets are so situated that they are not crossed by the orbits of swarms of meteors, and consequently they are undisturbed by any resisting medium.

The preceding investigations have all been based upon the assumption that the solar system is stationary. It is known, however, that the Sun is moving through space, and to be strictly accurate this movement should be taken into account. When it it is introduced the problem is greatly complicated, and the calculations are very long. Expressions have, however, been found for $\frac{da}{dt}$ and $\frac{de}{dt}$ for the values $p = 1$ and $p = 2$. They show in each case that the major axis is constantly diminished; but the eccentricity is sometimes decreased and sometimes increased, the sign of the variation depending upon the position of the orbit with respect to the direction of the Sun's motion. Special investigations would have to be made for the other elements also, and it is very probable that the position of the orbit plane in this case would not be stationary. The direction of the disturbing force is not always along the tangent to the orbit, and not always in the orbit-plane.

In conclusion a word may be said with regard to the generality of the investigations which have been made. It is true that they were based upon an assumed law of resistance, but this law is very general because it includes every case where the resistence varies directly as some power of the velocity. If the resistence is expressed by a series, then the right member of each of equations (3) becomes the sum of a number of terms similar to the one already involved, but containing different values of p and q. The general term, for p and q any numbers, has already been integrated, so that it is only necessary to apply this result to integrate the whole expression. In whichever way the resistance is expressed, the form of the equations allows us to make three general statements which hold always if the system is stationary in space. (1) The position of the orbit plane is unchanged; (2) the direction of the major axis, and the mean epoch, oscillate; and (3) the major-axis constantly diminishes. The varia-

tion of the eccentricity is secular, but nothing further can be said about it until the law of resistence is given. These are the results which can be called general. Further investigation awaits the demonstration of some definite law.

THE UNIVERSITY OF CHICAGO.

January 11, 1898.

THE VARIABLE STAR U PEGASI.

EDWARD. C. PICKERING.

Much difficulty has been experienced in determining the nature of the variations in the light of this star. Mr. Chandler states that he at first supposed that it was a star of the Algol type having a period of $2^d.06$ or $2^d.07$. It was then observed by Mr. Yendell who confirmed the variability and still regarded it as of the Algol type, but with a period of $0^d.69$. Mr. Chandler, under date of October 26, 1895, announces in the *Astron. Journ.* 15, p. 181, that the period is $5^h\ 31^m\ 9^s.0$ which "is probably only a moderate fraction of a second in error," that it is not of the Algol type, but that the times of increase and decrease are equal. Mr, Yendell (*Astron. Journ.* 16, p. 78) states that the time of increase varies from $1^h\ 28^m$ to $3^h\ 41^m$. Again Mr. Chandler (*Astron. Journ.* 16, p. 107) announces that the period is $5^h\ 32^m.25$, that it is perfectly regular, and that previous discrepancies are due to a large subjective error amounting to $0^m.6p$, in which p is the parallactic angle. The correction for this error will sometimes increase and sometimes diminish the observed time of minimum by as much as half an hour. He also states that the decrease in light is more rapid than the increase, and, referring to the short period variables η Aquilae and δ Cephei that "we may therefore regard U Pegasi, provisionally, as a type of variability distinct from this class of stars, as it evidently is from those of the Algol type."

Owing to these uncertainties it seemed desirable to determine the true form of the light curve photometrically, especially as such observations are free from the subjective error mentioned above, since the images compared are constantly interchanged. Assuming the light curve to be constant, it appeared possible to determine its form from observations made during a single evening. Accordingly, measures were made by Mr. O. C. Wendell with the polarizing photometer (*Astrophys. Journ.* II, 89) at-

tached to the 15 inch equatorial telescope of the Harvard College Observatory. The results for a single evening, December 28, 1897, are contained in the annexed table, the first column giving the Greenwich Mean Time, and the second the photometric magnitude found by adding the observed difference in magnitude between U Pegasi and the comparison star, + 15°.4916, to 8.90, the photometric magnitude of the latter star. These observations are also indicated by the crosses in the figure. The third column gives the residuals found by subtracting from the magnitude given in the second column that derived graphically from all the observations.

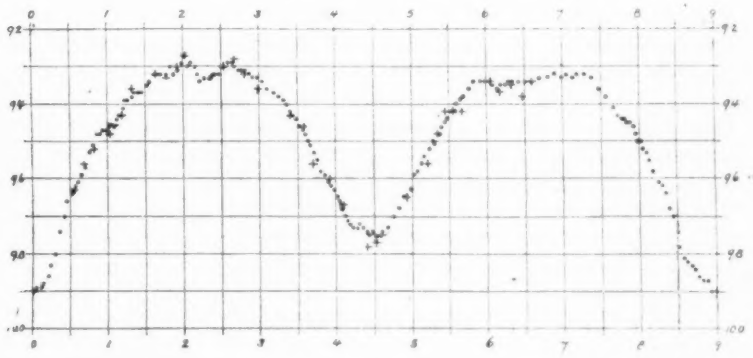
OBSERVATIONS ON DECEMBER 28, 1897.

G.M.T.		Magn.	O—C	G. M. T.		Magn.	O—C
h	m			h	m		
10	26.9	9.63	.00	13	48.7	9.60	−.01
10	34.2	9.56	−.01	13	58.6	9.67	−.02
10	42.2	9.52	+.01	14	17.5	9.78	+.04
10	55.3	9.48	+.02	14	24.5	9.77	+.02
11	4.7	9.43	+.02	14	34.0	9.75	+.02
11	12.4	9.36	−.02	14	43.0	9.75	+.07
11	31.3	9.32	−.01	14	50.0	9.65	.00
11	40.0	9.33	+.02	15	7.0	9.56	+.03
11	48.6	9.30	.00	15	15.0	9.48	.00
11	55.4	9.27	−.02	15	21.0	9.42	−.02
12	25.8	9.30	.00	15	26.8	9.42	.00
12	33.4	9.28	−.02	15	33.2	9.42	+.04
12	41.2	9.32	+.01	15	56.9	9.34	.00
12	51.1	9.36	+.03	16	3.0	9.37	+.02
13	17.4	9.43	+.01	16	13.3	9.35	+.01
13	27.2	9.46	−.01	16	21.2	9.38	+.04
13	36.9	9.56	+.03	16	29.6	9.34	.00

Each observed magnitude is derived from the mean of four sets of four settings each, the photometer being reversed between the second and third sets, and the positions of the images of the stars interchanged after the second setting of each set. The magnitude derived from each pair of sets differ on the average of ± 0.04 magn. The average values of this quantity during the seven successive hours of observation were 0.06, 0.03, 0.04, 0.03, 0.04, 0.04, and 0.04 respectively. As they show no progressive increase it is evident that the fatigue of the observer did not sensibly affect the accuracy of the observations. The accuracy is also unaffected by the altitude which was only 5°.6 at the end of the series. No correction has yet been applied for differential absorption, which is small but sensible, although the distance between the stars compared is only 15′. The computed probable error of four settings from these differences is ± 0.024 magn. The average value of the residuals in the third column is ± 0.017, only one being greater than 0.04.

It is obvious from an inspection of these observations that the star had nearly the same brightness after an interval of $4^h\ 30^m$, at first indicating that the variable had this period and not $5^h\ 31^m$. But later observations showed that the phenomena were not quite so simple. Assuming the period $4^h\ 20^m$ it soon appeared that the even minima were fainter than the odd. Observations continued through even minima on four nights gave the minimum brightness of U Pegasi, including errors of observation 9.94 9.94, 9.92, and 9.85 respectively, while corresponding measures of odd minima gave the magnitudes 9.78, 9.78, and 9.75. It thus became evident that the period should be doubled, and plotting the observations a light curve was formed closely resembling that of β Lyrae. The formula J. D. 2413514.6157 + 0^d.37478E has been adopted which closely represents the observations recently made and also a few less accordant observations made here in 1895. Changing the period by two seconds, to 0^d.3748, would alter the times of minima in 1895 by about an hour while the uncertainty is only a small fraction of this amount. The period may, therefore, be assumed to be $8^h\ 59^m\ 41^s$. The magnitude at maximum is 9.30, at primary minimum 9.90, and at secondary minimum, 9.75.

PHOTOMETRIC OBSERVATIONS.



The above figure represents the observations on a scale such that in the ordinates one division corresponds to a tenth of a magnitude, and in the abscissas to 30 minutes. All the observations made by Mr. Wendell on eight nights are included, none being rejected for discordance. In fact, the most discordant set of sixteen settings differs from the mean curve by less than a tenth of a magnitude. Each dot represents 80 settings, the method of

overlapping means, in common use in meteorology, being employed. The observations of December 28, as stated above, are represented by crosses, each representing the result of sixteen settings. The total number of settings was 2784 and the entire time of observation including rests, about 30 hours.

If we neglect the difference between the primary and secondary minima, reduce the half period to fractions of a day, and multiply it by 16 we obtain the product $2^d.99824$, or very nearly three days. Therefore, the phases recur at nearly the same times every three days. If we multiply the period $5^h\ 32^m\ 15^s$ by 13 we obtain $2^d.99948$, or very nearly the same quantity. In either case, therefore, the phases would recur at the same times every three days, but with the second period the interval between successive minima would be greater than with the first period in the ratio of 16 to 13 If then we used the second period we should find the observed minima in error by an amount equal to three-thirteenths of the hour angle, or expressed in minutes $0^m.92h$, in which h is the hour angle expressed in degrees. Since the latitude of Cambridge is $+\ 42°.4$ and the declination of U Pegasi is $+\ 15°.4$ it follows that when h is small we may write $h = 0.61p$ and $0^m.92h = 0^m.56p$ which agrees very nearly with $0^m.6p$ the correction found empirically by Mr. Chandler from his observations. The coefficient becomes larger with large hour angles having the values 0.61, 0.74, and 0.91 for $h = 15°$, $30°$, and $45°$, respectively. It, therefore, follows that by the application of the correction $0^m.6p$ not only were the observed times of minima changed in some cases by more than half an hour but an error of more than an hour was introduced into the half period. An arbitrary correction proportional to the parallactic angle is not to be recommended, since by assuming different values of the coefficient, very different values of the period will be found.

From the above discussion it appears that U Pegasi is no longer the variable star having the shortest period known. This position appears to be held by the variable ω Centauri 19, discovered by Professor Bailey who finds its period to be $7^h\ 11^m$. Although U Pegasi can no longer be regarded as an example of that peculiar class of short period variables having a single maximum in which the decrease is more rapid than the increase, this class is still represented not only by S Antliae, but by ω Centauri 24, which Professor Bailey finds to decrease twice as rapidly as it increases, while ω Centauri 45 increases at least five times as fast as it diminishes.

January 14, 1898.

Harvard College Observatory Circular No. 23.

ON THE DEFINITION AND INTENSITY OF A STAR'S IMAGE IN THE FIELD OF VIEW OF A PARABOLIC REFLECTING TELESCOPE.

J. M. SCHAEBERLE.

FOR POPULAR ASTRONOMY.

In the February number of "POPULAR ASTRONOMY" Professor Wadsworth has two articles which call for a brief reply, since silence on my part would be construed to indicate that I consider his criticisms sound.

In the first paper he states, without qualification, that my results for the mirror which I call worthless "are grossly in error." A disinterested consideration of the problem would, I feel sure, decide that my view of the case is correct.

My argument is as follows: If the field of view of such a telescope, having the angular aperture given on page 508 of this journal, is examined with an eye-piece so placed that the focal image of a star situated exactly on the optical axis of the paraboloid (of which the mirror's surface is a part) is central and well-defined, all the other star images in the same field of view will be increasingly distorted as the angular distance from the optical axis increases.

By shifting the eye-piece towards, or away from, the mirror some of the images will grow less distorted at the expense however of the destruction of the focal image and an increased distortion of images in certain other parts of the field of view, so that *all* the images will now be practically worthless.

According to my view of the case there is only one proper position for the eye-piece, and for this one position there will be only one point in the field of view which is sensibly free from distortion. The remainder of the field of view will be distorted to a degree indicated by my previous statements, viz: *The image of a star at an angular distance of 5′ from the optical axis would have a radial aberration of about three-quarters of a minute of arc.*

If an experienced observer were forced to use such an instrument he would try to bring each star-image to be examined as near to the true focal point as possible by moving the whole telescope*. He would not think of changing the focus for each dis-

* The slightest deviation of the image from the true focal point would cause enormous variations in the *intensity* of the light of the star's image. So that for this reason alone there would be serious objections against doing even spectroscopic work with such an instrument.

torted image to obtain, in the end, only an image which was still greatly distorted though in a somewhat less degree.

A photograph of the field of view, with the plate included at the most favorable angle, would be of no value. Compared with the results to be obtained with the ordinary form of mirror I call the proposed instrument worthless. Dr. Poor, the investigator who is most interested in this form of telescope, is now of the same opinion.* It is Professor Wadsworth's privilege to think differently.

The second paper illustrates very strongly how fallacious certain conclusions may be which are drawn from hasty or biased examinations of seemingly confirmatory evidence derived from actual observations.

Professor Wadsworth lays great stress upon the fact that Herschel discovered the inner satellite of Saturn when it was less than 10″ from the ring.† Yet Herschel himself states that at the time of discovery *the ring was barely visible in the 40-foot telescope.*

To detect Mimas in a four-foot telescope should certainly not be called "an observation of extreme delicacy."

During the past twenty-five years I have frequently made *front-view* experimental observations on celestial objects with circular areas of parabolic mirrors in which the line drawn from the *undistorted, well-defined* image to the centre of the reflecting surface was inclined more than 3° to a normal to the surface at the same point; the ratio of focal length to aperture being even less than 7 to 1. Nevertheless all these observations were made not 3° from the optical axis but in the immediate vicinity of this axis.

Being fully aware of Herschel's great skill in making reflecting telescopes and his remarkable powers as an original investigator, I do not believe that his observations—or those of his son later on—were made between 1° and 2° from the optical axis. From my own experience in making parabolic reflectors I feel convinced that the immortal Herschel in figuring his great mirror finally obtained by repeated trials a surface for which the *optical* center was just outside of the edge of his mirror. That it was not long before he discovered the fact that the definition was but on a

* See *Astronomical Journal* No. 421.

† Professor Wadsworth here erroneously assumes that the maximum distortion as given by my formula represents the size of a *disk* over which the light is uniformly distributed, when in fact the image is a long blurred, wedge-shaped one with a drawn out nucleus the brightest point of which is near the vertex of the wedge, where only the light from the central areas of the reflector comes to a focus.

normal passing through a point in or beyond this edge, can, I think, be taken for granted. A star image at the focus of a properly constructed *front-view* telescope will necessarily be free from distortion, even when the true focal point is wholly exterior to the cylinder of direct rays which go to form the focal image.

Professor Wadsworth's next effort to prove that my results are incorrect is a reference to his measures of printed star images to be found in the published reproductions of the excellent photographs of Roberts. Instead of using bright stars (which, aside from the inevitable spread of the photographic image, should show a distortional effect) he selects faint ones and finds that the impressions "more than 1° from the center" certainly do not exceed 5"! What he really measured was in fact nothing more than the brightest part (formed by the more central area of the reflector) of the distorted star image which alone survived the effects of the successive manipulations necessary for transferring the image from the telescope to the printed page.

In *A. J.* 413 I stated that the star image increases in size *along a line drawn from the optical centre*, that is, the image is wedge-shaped, the vertex of the wedge being on the side towards the optical axis.

For myself, the images on every full plate of Robert's valuable publication indicate that my theoretical results are correct and I feel confident that a disinterested party examining these same plates* will come to a like conclusion.

How greatly the visible extent of the fainter portions of the distorted star image may be diminished in the photograph, and still more so in a published print, may be gathered from a simple practical example.

The photograph of a comet having a tail several degrees in length can be made to vary all the way from a star-like image, representing the brightest part of the nucleus, up to an elongated area similar in form and general extent to the visual image.

Measures of the character described by Professor Wadsworth are plainly not to be considered as final, to say the least.

"In the case of the parabolic mirror by far the larger proportion of the light which unites to form an image of a point source, comes from the outer portion of the mirror." With this state-

* The optical center of the plate does not always coincide with the geometrical center. On some of the plates, also, the errors of guiding must be allowed for. It is also probable that the mouth of the telescope-tube acts as a diaphragm to cut off certain injurious rays greatly inclined to the optical axis; as is the case for the Crossly telescope where the radius of the mouth of the tube is less than one inch greater than the radius of the mirror. The length of the tube is 16 feet.

ment I of course agree; but continuing, Professor Wadsworth makes a hasty and wholly erroneous claim * in which he clearly implies that the outer zones have the preponderating influence in the formation of an image not on the optical axis, and that the effect of the inner zones is of a secondary nature. He says: "To attach equal importance to the effect of the inner and outer zones in determining the prejudicial effect of aberration is therefore fundamentally wrong." In fact however these outer zones are the very ones which tend most to destroy the definition, as I shall now demonstrate by means of an experiment (with a sensibly perfect paraboloid) which fully confirms the correctness of my theoretical results.

An annular ring having an external diameter of 34 inches and an internal diameter of 12 inches was cut from a sheet of heavy card board and fastened centrally to the cell of the Crossly reflector by means of 4 narrow radiating arms, the ring being within half an inch of the mirror's surface. The telescope was then directed to Rigel (β Orionis) and on examining the images of the two circles formed either inside or outside of the true focal plane I found 1st, that of the several eye-pieces available only one, that giving the highest power, gathered the light from the extreme outer zone: 2d, The radius of the whole field of view for full aperture of the telescope (found by shifting the eye-piece over the whole focal plane and noticing the limiting positions of the eye-piece) was found to be considerably less than half a degree, on account of the small diameter of the mouth of the tube which acts as a diaphragm. (The limited size of the diagonal mirror also serves to cut off the most injurious of the inclined rays.) Now although the amount of light reflected from the zone 2 inches in width at the circumference of the mirror was actually more than twice † as great as the amount reflected from the 12 inch circle minus the area occulted by the 9-inch flat. Still, for all those positions in which the *two* focal images formed were distinctly separated from each other, the one nearer the optical axis formed by the central area—was intrinsically *very many times brighter* than the large blurred image (which could not be brought to a point by shifting the eye-piece) formed by the outer reflecting area. *None of the light from the outer area assisted in*

* A fundamental principle of optics is here involved which Professor Wadsworth still overlooks even after his attention has been called to the matter.

† In the first trial of the annular ring the outer diameter was 34 inches but the blurred image of α Urs. Min. (the Star used) formed by the outer area was so faint and indefinite compared with the bright and fairly well-defined stellar image formed by the central area that I doubted the width of the outer reflecting ring.

forming the fairly well-defined stellar image formed by the central area, as was readily demonstrated by slowly throwing the eye-piece out of focus causing both rings to reappear, each starting from its own focal image.

The companion* to Regel as formed by the central area was well seen whenever it was at a sufficient angular distance from the blurred image, as the eye-piece (kept near the edge of the available field) was brought into different position-angles with reference to the optical center of the field.

About 20′ from the optical axis (at the edge of the available field of view for full aperture) the most distant easily visible part of the blurred image from the center of the fairly well-defined image was plainly farther away than the distance between Rigel and its companion; while the seemingly clear space between the two images (*viz*: the distance from the nearest point of the blurred image to the center of the small bright one) was about one-third of the companion's distance.

Calling the companion's distance 9″, the maximum distortion actually observed (only 20′ from the optical axis) was greater than 10″; a value in practical agreement with the result given by my formula.

The phenomena accompanying a displacement of the image with reference to the focal point are so marked in this telescope that the appearance of a bright star, when it is only about one minute of arc from the focal point, will indicate the direction of the optical axis; and at large angular distances the field of view close to the star, but on the side away from the blurred image is strikingly free from disturbing rays, owing to the fact that this side of the image is formed by only those rays which come from a comparatively small central area of the reflector (equivalent to a telescope of small angular aperture); so that for full aperture a close and moderately faint companion in this region might be seen even better than if the images were nearer the optical axis.

The observed distribution of light in the distorted image for full aperture is sensibly in agreement with that given by the geometrical theory. If the actual intensity of the light at any given point of the image should be required with extreme rigor, it would be necessary to consider the principles of the wave-theory in connection with the geometrical treatment.

In this telescope, at a distance of 30′ from the optical axis (and

* The *blurred* image of the *companion* could not be recognized even in the most favorable position. So far, therefore, as the visual observation alone was concerned, the action of the central area was almost an infinite number of times more effective than that due to reflection from the outer ring having twice the area.

without a diaphragm of any kind) it is certain that more than two-thirds of all the stellar light coming from the whole mirror is thrown into a most harmful blurred area, and that less than one-third of the total light goes to form the image which can be said to represent the star, and this image is formed only by the light reflected from the central area of the mirror.

Aside from the harmful effects of the blurring, the intensity of a star image as measured by the *effective aperture* (without a diaphragm) of a parabolic reflector may be said to decrease as the angular distance from the optical axis increases.

Sensibly perfect definition and maximum intensity of the image, for full aperture, is only to be obtained exactly at the focal point.

For very large angular apertures a slight deviation from the true focal point becomes sensibly apparent.

In conclusion I desire to state that my approximate expression for the *blurring factor* is derived directly from a theoretically rigorous formula (see *Astronomical Journal* No. 423) depending upon the laws of geometrical optics.

Feb. 15, 1898,

LICK OBSERVATORY, University of California,
Mount Hamilton, Cal.

ON THE ABERRATION OF PARABOLIC MIRRORS.

J. M. SCHAEBERLE.

"In his article in *A. J.* 420, Dr. Poor makes some statements which call for remarks from myself.

First, to make plain that which is "obscure" in my formulas, let what I have elsewhere called the *blurring factor* be given by the expression,

$$(1) \qquad \frac{\text{Dist. from focus to farthest point of image}}{\text{Dist. from focus to nearest point of image}}$$

all distances being measured in the focal plane. Using my own notation, *A. J.* 413, we can at once write the *rigorous* equations.

$$(2) \qquad \text{Greatest distance} = \rho \tan \theta \sec (v + \theta)$$

$$(3) \qquad \text{Least distance} = F \tan \theta$$

Consequently,

$$(4) \quad \text{Blurring factor} = \frac{\rho}{F} \sec (v + \theta) = \frac{1}{\cos (v + \theta) \cos^2 \frac{1}{2} v}$$

When θ is small we can write, in place of the *theoretically rigorous* expression (4), the approximate equation

$$(5) \qquad \text{Blurring factor} = \frac{1}{\cos v \cos^2 \frac{1}{2} v}$$

which, for the ordinary purpose of illustrating the magnitude of the errors to which I have called attention, is sufficiently approximate.

To show how slight the difference is between the results given by my rigorous and approximate formulas, even for large values of θ, I have, as an example, computed the following values of the angular amount of the blurring for different ratios of focal length to aperture, for $\theta = 30'$.

Ratio of Focal Length to Aperture	Rigorous Formula	Approximate Formula
	"	"
5	15.1	13.5
6	10.4	9.2
7	8.1	7.0
8	6.3	5.4
10	4.3	3.4
14	2.3	1.4
18	1.6	1.1
22	1.1	0.7

Dr. Poor is in error when he assumes that his formulas (which have doubtless been derived since the publication of my investigation in *A. J.* 413) are general and exact; they are exact only for the single special case of aberration in the plane containing the optical axis and the point whose image is under consideration. In like manner my simple *rigorous* formula (4) gives only the aberration along a line drawn in the focal plane from the focal point.

The aberrations in other directions are always less, reaching a minimum, both in the spherical field and focal plane, in a direction at right angles to a line drawn from the image to a focal point.

Dr. Poor is therefore in error in his "*Conclusion*" when he states that "The image of a faint star situated at an angle α from the center of the field will be a disk whose diameter is γ," etc.; for his γ is simply the maximum length of the elongated star-image, whose form, even in the spherical field, is more nearly linear than it is circular.

As to the method of derivation of my formulas I wish to say that, as the photographic plate for a symmetrical reflector must be normal to the optical axis, and also contain the focal point, it seemed to me superfluous to first deal with the blurred and distorted image in the spherical field.

The extreme simplicity of formula (4), which is derived directly by considering only intersections with the focal plane, is only rivaled by its theoretical exactness."—*Astronomical Journal*, No. 423.

LICK OBSERVATORY, University of California.
Mount Hamilton, Cal.

ON NEWTON'S LAW OF GRAVITATION.*

PROFESSOR H. SEELIGER.

FOR POPULAR ASTRONOMY.

II.

As was above stated, I pass here, to the more detailed consideration of the question, whether the number of luminous bodies in the universe is also to be regarded as infinitely great, and what conclusions as to the brightness of the celestial vault, are connected with the question. We have to do essentially, with a subject which Olbers discussed in a well-known and oft-cited pamphlet.†

Olbers says in another place: "If suns be present in infinite space, whether located at approximately equal distances from each other or distributed throughout space in galactic systems, their masses will nevertheless be infinite and therefore, the entire celestial vault would be as bright as the Sun, for every line which I conceive drawn outward from my eye would necessarily strike a *fixed* star, and then every point on the sky must send to us stellar light, or what is the same thing, sunlight."

Similar considerations were also announced much earlier by the Lausanne astronomer, L. de Cheseux, as W. Struve‡ has pointed out.

Olbers' course of reasoning is the following: If the assumption of the existence of infinitely numerous *shining* cosmic bodies be granted, and this assumption is just as little to be doubted as the inference following from it, that light, transmitted through space, suffers a loss of intensity, *i. e.* that an absorpsion of light exists in space, similar to that which the most transparent media exercise, all difficulties will be removed.

On the other hand, this assumption certainly cannot be granted as a necessity, perhaps not even as a plausible hypothesis.

* Continued from page 551.

† Ueber die Durchsichtigkeit des Weltraums. *Ast. Jahrbuch* for 1826. Olbers' Works, Vol. I, p. 133-144.

‡ Etudés d'Astronomie stellaire p. 34.

As has been already remarked, the mass contained in the universe, which exerts the force of gravitation, cannot be identified with the self-luminous part of the same, at least not without inquiry; and even if we are disposed to regard the former as infinitely great, it by no means follows from this alone that the same holds for one of its parts.

Let us first derive the expression for the average superficial brightness of the celestial sphere. This expression results quite easily from considerations, I have worked out for a more complicated case.* Let $d\varepsilon$ denote a self-luminous element of surface situated anywhere in space and dq', the quantity of light it sends out to the observer, if its light be unobstructed and its inclination to the sight-line be 90 degrees. This element may however, be wholly covered by bodies lying before it which, for the sake of simplicity, will here be considered spheres. In this case the quantity of light sent to the observer is of course equal to 0. On the average the superficial element, $d\varepsilon$, will transmit to the observer a definite quantity of light, which may be represented by dq and we shall have:

$$dq = dq' \cdot \omega$$

where ω represents the *probability* that the element $d\varepsilon$ is not occulted by any intervening sphere. ω may therefore, always be regarded as a proper fraction characterizing the *mean value* which is sought. Since we may conceive of a finite space bounded in any way whatever, let us suppose that in all space P there exist N_1 spheres of radius ρ_1, N_2 with radius ρ_2, etc. N_n spheres with a radius ρ_n. Let also ω_m represent the general probability that no occultation occurs through the sphere of radius ρ_m, then we have

$$\omega = \omega_1 \, \omega_2 \, . \, . \, . \, \omega_n$$

We may conceive the surface of each luminous sphere composed of such elements as $d\varepsilon$. If then, q_m denote the original and unobstructed light of a self-luminous sphere of radius ρ_m, such a sphere will send to us on the average, a quantity of light represented by

$$q = q_m \cdot \omega \tag{1}$$

provided ρ_m be made sufficiently small. If J_m denote the mean light intensity (*i. e.* the quantity of light which the superficial element unity, would send to an observer at distance unity, with its surface perpendicular to his sight-line) of a sphere K_m, of radius ρ_m, and further, r denote its distance from the observer, then we have:

* Theorie der Beleuchtung staubförmiger Kosmischer Massen. Abhandlungen der Münchener Akadamie der W. Vol. XVIII, 1893.

(2) $$q_m = J_m \cdot \pi \cdot \frac{\rho_m^2}{r^2}$$

J_m might of course, be zero. This would be the case with a dark body. ω_m is the probability that the center of K_m lies anywhere within the space P, though not within a right circular cylinder, C_m, of radius ρ_m with an axis extending from the observer to the body. We may now assume, that cosmic bodies are quite sparsely distributed throughout space, a value may then be quite easily and accurately assigned to ω_m. If $d\tau$ denote the volumetric element and φ_m, a function of the space coördinates, denoting the relative frequency of the occurrence of a sphere K_m, in the separate portions of space, we may then write

(3) $$\omega_m = (1 - X_m)^{N_m}$$

where $$X_m = \frac{Y_m}{Z_m};\ Y_m = \int_{C_m} \varphi_m d\tau;\ Z_m = \int_P \varphi_m d\tau.$$

The integral Y_m is to be extended throughout the cylindrical space C_m and Z_m throughout the entire space P. If $d\omega$ denote the opening of a cone whose vertex is at the observer, we shall have:

$$d\tau = r^2 d\omega dr,$$

Within this element is situated:

$$\frac{N_m}{Z_m} \cdot \varphi_m r^2 d\omega dr$$

spheres of radius ρ_m, and the amount of light sent to the observer will therefore be,

(4) $$\sum_1^n {}_m N_m \frac{\varphi_m}{Z_m} \cdot q_m \omega r^2 d\omega dr.$$

Integrating now, with respect to r, from $r = 0$ to $r = R$, where R denotes the limiting radius of the space P, and dividing by $d\omega$, we shall obtain the average superficial brightness in a direction corresponding to the direction-angle occurring in function φ_m. We shall also obtain the mean brightness of the entire heavens if we integrate "(4)" first with respect to r and then with respect to $d\omega$ over the entire surface S of the unit sphere and divide the result by 4π. We have then

$$\text{(I)} \qquad h = \frac{1}{4}\int_S d\omega \int_0^R dr \sum_1^n {}_m N_m \frac{\varphi_m}{Z_m} \cdot J_m \cdot \rho_m^2$$

$$(1 - X_1)^{N_1} \cdot (1 - X_2)^{N_2} \cdot \ldots \cdot (1 - X_n)^{N_n}$$

To obtain convenient expressions for computation and which are at the same time, accurate enough, we need only consider that X_m is very small, whereupon $(1 - X_m)^{N_m} = e^{-N_m X_m}$ and if this be introduced into (I) we have

$$\text{(II)} \qquad h = \frac{1}{4} \cdot \sum_1^n {}_m \frac{N_m}{Z_m} \cdot \rho_m^2 \int_S d\omega \int_0^R dr \cdot \varphi_m J_m e^{-\sum_1^n {}_m N_m \frac{Y_m}{Z_m}}$$

Let us first apply this formula to the simplest case, in which there exists a uniform distribution of spheres in space. We have then

$$\varphi_m = 1 \text{ and } Y_m = \rho_m^2 \cdot \pi \cdot r$$

We may assume the space P limited in any desired way, and then cause this limit to recede infinitely. Let, for example, a sphere of radius R be assumed for P. Therefore:

$$Z_m = \frac{4}{3}\pi R^3.$$

Formula (II) becomes now:

$$h = \frac{3}{4} \cdot \int_0^R dr \sum_1^n {}_m N_m \frac{\rho_m^2}{R^3} \cdot e^{-\frac{3}{4} r \Sigma N_m \frac{\rho_m^2}{R^3}} \cdot \frac{1}{4\pi} \cdot \int_S J_m d\omega.$$

The average superficial brightness J of the spheres, may be defined by

$$J = \frac{1}{4\pi}\int_S d\omega \cdot \frac{\Sigma N_m \rho_m^2 J_m}{\Sigma N_m \rho_m^2}.$$

Consequently, we obtain,

$$h = \frac{3}{4}\int_0^R J \cdot dr \cdot \frac{\Sigma N_m \rho_m^2}{R^3} \cdot e^{-\frac{3}{4}\frac{r}{R^3}\Sigma N_m \rho_m^2}$$

If now we put,

$$\frac{3r}{4R^3} \cdot \Sigma N_m \rho_m^2 = x; \quad \frac{3}{4}\Sigma N_m \left(\frac{\rho_m}{R}\right)^2 = \Gamma$$

we have finally:

(5) $$h = \int_0^{\Gamma} J \cdot e^{-x} dx.$$

Introducing now, a mean radius ρ_0, where

$$\rho_0 = \frac{\Sigma N_m \rho_m^3}{\Sigma N_m \rho_m^2}$$

and ρ_0 will lie between the greatest and the least values of ρ_m. Let the sum of the volumes of the spheres contained in the space P, be designated by K, then

$$\frac{K}{P} = \frac{1}{R^3} \cdot \Sigma N_m \rho_m^3 = \frac{\rho_0}{R} \Sigma N_m \left(\frac{\rho_m}{R}\right)^2$$

and for simplicity, we may put

$$\lambda = \frac{3}{4}\left(\frac{K}{P}\right) \cdot \frac{r}{\rho_0}; \; \Gamma = \frac{3}{4}\left(\frac{K}{P}\right)\frac{R}{\rho_0}.$$

If we assume, as was done in the first part of this paper that the matter in the universe is distributed in such fashion as to have finite, though arbitrarily small, density, then K/P is finite and T may be made as great as desired by increasing R. Γ must then, for the entire universe, be put equal to infinity. If J were independent of λ, h would equal J and in this hypothesis, the essence of Olber's result is contained. For the general case, a similar reduction may also be performed. If J_ω denote the mean value of the individual J_m's, in the direction ω,

$$J_\omega = \frac{\Sigma N_m \rho_m^2 J_m \varphi_m \cdot \frac{1}{Z_m}}{\Sigma N_m \rho_m^2 \cdot \varphi_m \cdot \frac{1}{Z_m}},$$

then according to (II)

$$h = \frac{1}{4}\int_S d\omega \int_0^R dr J_\omega \cdot \left(\Sigma N_m \rho_m^2 \varphi_m \frac{1}{Z_m}\right) \cdot e^{-\left(\Sigma N_m \frac{Y_m}{Z_m}\right)}.$$

and with precisely the same degree of precision as holds for equation (I), we may put;

$$Y_m = \rho_m^2 \pi \cdot \int_0^r \varphi_m dr.$$

Making

$$x = \pi \cdot \Sigma N_m \rho_m^2 \cdot \frac{1}{Z_m} \int_0^r \varphi_m dr; \; \Gamma = \pi \Sigma N_m \rho_m^2 \cdot \frac{1}{Z_m} \cdot \int_0^R \varphi_m dr,$$

we have

$$\text{(III)} \qquad h = \frac{1}{4\pi} \int_S d\omega \int_0^\Gamma J_\omega \cdot e^{-x} \cdot dx$$

It will perhaps be clearer here to give the mean superficial brightness h_ω, in a definite direction;

$$\text{(IIIa)} \qquad h_\omega = \int_0^\Gamma J_\omega \cdot e^{-x} dx.$$

This is precisely formula (5). In this general case it may also be shown that Γ increases indefinitely with R, if the matter distributed uniformly in the chosen direction has a finite density.

The number ν_m of spheres K_m in the volumetric unit is

$$\nu_m = \frac{N_m \varphi_m}{Z_m}.$$

If "n" be the entire number of spheres within the volumetric unit, and hence

$$n = \nu_1 + \ldots + \nu_n$$

and if ρ_0 denote the mean value of all the ρ_m's, we may put

$$\Sigma N_m \rho_m^2 \varphi_m \cdot \frac{1}{Z_m} = \Sigma \nu_m \rho_m^2 = n\rho_0^2.$$

We have also

$$\Gamma = \pi \int_0^R n\rho_0^2 dr.$$

If now, we denote the arithmetical mean by $M(n\rho_0^2)$, we have

$$\Gamma = \pi R M(n\rho_0^2).$$

From this formula results immediately the proposition stated above.

It is impossible to obtain h from observation, to any high degree of approximation. Since furthermore, J_ω can assume only finite values, we may represent the observed h_ω in finite values of Γ as accurately as may be desired.

What is here said of the brightness of the celestial vault, holds even more generally. By virtue of perspective occultations of the cosmic bodies, only a finite portion of space will be accessible to our perception and its dimensions will in fact, be materially limited by the masses of nebulæ which seem to surround us on all sides and limit the scope of vision.

We know as yet so little concerning the distribution of the stars

in space that we may use formula (5), which is of the same form as (IIIa) and obtain from an example, an idea of the possible numerical values involved.

Let us first consider Olber's question of the absorption of light in space. This may be done, as is at once evident, by putting into the formula $J_m \cdot e^{-\lambda r}$, instead of J_m, where λ denotes the hypothetical co-efficient of absorption. Formula (5) then becomes

$$h = \frac{3}{4}\int_0^R J \cdot e^{-\left(\lambda + \frac{3}{4}\frac{K}{P}\cdot\frac{1}{\rho_0}\right)r} \cdot \frac{K}{P\rho_0}\, dr. \tag{6}$$

If for example, J be assumed independent of r, there results for $R = \infty$,

$$h = J \cdot \frac{\frac{3}{4}\frac{K}{P}}{\lambda\rho_0 + \frac{3}{4}\frac{K}{P}}. \tag{7}$$

The numerical reduction of (7) can be made only quite roughly and upon hypotheses more or less arbitrary; because of our total ignorance of the mean values of the light-intensity, magnitudes, and distances of even those cosmic bodies nearest us. We can, in point of fact, do nothing more than ascertain the orders of magnitude of our various quantities. We shall take as our unit distance, the distance of a star whose parallax is 0″.2 and for brevity, this unit will be called "*Sirius-distance*." The mean parallax of stars of the 6th magnitude will be taken at its usual value of 0″.02. Within this value of R, there are said to be 6,000 stars distributed with a close approach to uniformity. ρ_0 will be taken as double the solar diameter. To this distance we have then

$$\frac{3}{4}\frac{K}{P} = 3.10^{-24}.$$

The absorption of light at the Sirius-distance may be taken as 0.1%. Also $\lambda = 10^{-3}$; $\rho_0 = 0.9 \times 10^{-8}$; $\lambda\rho_0 = 0.9 \times 10^{-11}$. If now, the same uniform density of distribution of the cosmic bodies of the entire universe be assumed, then according to (7)

$$h = J \cdot 3.3 \times 10^{-13}.$$

The surface brightness of the Moon is about 1/600000 that of the Sun. The brightness of the background of the sky at full Moon is according to Olbers,* about 1/100000 of that of full

* Olber's Works, p. 139.

Moon. This datum I am unable to check. If J_H denote this brightness and if we put $J = \gamma \, . \, J_0$ where J_0 is the solar brightness, we find

$$h = \frac{\gamma}{50} \cdot J_H. \qquad (8)$$

At all events, this example shows that absorption suffices to explain the low mean brightness of the sky and this is true, even if the number of luminous masses grows indefinitely, with the portion of space enclosed. This however, can occur only when the co-efficient of absorption exceeds a certain magnitude, for according to (7), h is small with respect to J only when $\lambda\rho_0$ is very great as compared with $3K/4P$. Furthermore, it is clear, that the assumption of an absorption is not *necessary*, for the same conclusion is reached if a certain mean value of J does not exceed a definte small value. Considering formula (5), we have

$$h = \int_0^\infty J \, . \, e^{-x} \, dx,$$

and we can make h small by various hypotheses concerning J. The simplest of these cases is that J is on the average very small. This would mean simply that everywhere in space, many dark or few bright bodies exist, in which case, very extended regions, either in our own neighborhood or in other parts of the universe might be excluded. With the above numerical values, there would result for example, for the brightness of the celestial vault, if to this brightness, all cosmic bodies to r' Sirius-distances, the assumed density of distribution contribute;

$$h = \int_0^{x_1} J \, . \, e^{-x} \, dx; \quad x_1 = 3.3 \times 10^{-16} \, . \, r.$$

If J be constant and r is not too great, we have

$$h = 3.3 \times 10^{-16} \, . \, r \, . \, J.$$

If r be put equal to 1000, we obtain the numerical values of (8). From such reflections as these, it follows, that we need only assume the light-intensity of cosmic bodies to be on the average, even with enormous distances, very small. This latter assumption holds also for finite distances, for by reason of the factor e^{-x}, it is of no consequence what form the mean light-intensity may have for very large values of r. Assumptions of this sort seem to me to be not only entirely permissible, but even preferable to any others.

The time during which a fixed star may be in such condition as to emit rays both visually and photographically effective, must be regarded as extraordinarily short compared with the interval during which it does not possess this capability. If cosmic bodies in the most varied stages of development are distributed throughout space after the law of chance, their mean brightness in all parts of space would necessarily be very low. But it is a matter of serious doubt, whether the hypothesis of accidental distribution is tenable. Nevertheless, it is not to be overlooked that this luminous intensity at any stated epoch, even on the assumption of a contemperaneous beginning of the developmental process, must depend very essentially upon the size, power of heat conduction, and other physical characteristics of the individual bodies. Among these physical peculiarities must of course, be included such facts as, that smaller bodies cool in general, more quickly than larger. Many facts of experience indeed, indicate that large bodies relatively near us, shine with very different intensities. Even for those portions of space near us, there is substantially no objection whatever to the belief that the number of dark stars far exceeds that of the bright. But even this assumption is far from necessary. The stage of development attained by any cosmic body is a function of both time and place. The different portions of space will accordingly, at any definite instant, be in very different conditions of growth, and from what has been said in the foregoing, we must assume, that by far the larger number of these conditions are not favorable to luminosity.

We must also consider the light-time *i. e.* the time consumed by light in making the journey from the bodies of space to us. For very distant regions then, these intervals of time must be considered in the problem of cosmic development. It is however, easy to see that the conclusions of the foregoing arguments are, in no material respect to be modified.

But in whatever way these considerations may be followed out or altered in their details, it must at any rate, follow from them, that the fact of the low general brightness of the sky does not necessarily indicate the existence of an absorption of light in the sense in which Olbers argued for it.

(CONCLUDED).

PLANET NOTES FOR MARCH.

H. C. WILSON.

Mercury will be at superior conjunction, behind the Sun, March 16, and will be hidden by the glare of the Sun during the most of the month.

THE CONSTELLATIONS AT 9 P. M., MARCH 1, 1898.

Venus is now evening planet but will be too nearly in line with the Sun to be easily seen during this month. March 26th, at 8 A. M. Central standard time, Venus and Mercury will be in conjunction in the constellation Pisces, Mercury being a litttle over a degree north of Venus at that time.

Mars is yet pretty close to the Sun to be seen well. It is moving northeast-

ward from Capricorn into Aquarius (see chart p. 494 Jan. 1898) and is visible only a little while in the morning.

Jupiter may be seen toward the eastern horizon about 8 P. M., but one will have to wait until about midnight for good views of the planet. For the course of Jupiter among the stars see the chart on page 494 in the January number of POPULAR ASTRONOMY. No reports of observations of the planet this year have reached us, yet doubtless many have begun systematic work in studying the markings of the planet and the phenomena of the satellites. Jupiter will be at opposition March 25.

Saturn may be observed in the morning, being near the meridian at 5 o'clock. His declination is 20° 26′ south, so that his altitude is low, even when on the meridian, and no very fine views may be expected by northern observers. Still, since the rings are opened to nearly their greatest width, the observer will obtain a great deal of satisfaction from the study of the planet. Saturn is in the constellation Scorpio about 7° northeast of the red star Antares. The planet will be at quadrature 90° west of the Sun, on the morning of March 2.

Uranus was at quadrature 90° west from the Sun, April 21, and may be found in the same constellation with Saturn, about 10° west from the latter, near the star β Scorpionis. Its dull green disc can readily be recognized with a telescope of moderate power.

Neptune will be at quadrature, 90° east from the Sun, March 10, and so is best observed early in the evening. It is to be found about 4° west from the star ζ Tauri, among the telescopic stars west of Omicron Tauri.

Sunspots.—Another great disturbance of the solar surface came into view between Feb. 7 and 12. On the 12th, 15th and 17th this group contained 3 large and many small spots. Feb. 21 this group had disappeared by rotation, but another considerable group, first seen Feb. 15, was near the center of the Sun's disc.

A Few Facts About the Recent Sunspot.—The recent large sunspot was chiefly remarkable as a reaction after a period of quiescence that has not been equalled in more than six years past. For two weeks in October and for twelve days in November last not a single spot was visible in a four-inch lens. Between these periods of unspottedness and also afterwards an occasional marking appeared, but until Dec. 6th they were insignificant in size, faint in tint, and of short duration.

On the morning of that date a large group was inside the northeast limb, and became visible to the naked eye in a few days when the view was less foreshortened. Though large for this period so near the minimum of solar disturbance, it was far surpassed by the enormous sunspots of Feb. 4th 1892, and of August 5th, 1893 and even by several of lesser size that appeared before and since those dates. Among them may be mentioned those of August 1895, of Feb. and of September 1896, and of January 1897. In these the components were more compact, whereas the recent group distributed over an area more than 150,000 miles in length had immense tracts of white surface intervening. The chief component was about 27,000 miles in diameter and a few were only penumbral markings. When central it extended from 8 degrees to 15 degrees heliographic latitude north, which is one of the most prolific of sunspot zones. The drawings show the changes that took place when near the centre of the disc. From the 13th it

commenced to diminish in size, and in a four-inch telescope it was not discernable after the 18th inst. when it was close to the west limb.

San Francisco, Dec. 30th. ROSE O'HALLORAN.

Occultation of the Pleiades January 30th, 1898.—The occultation of some stars in the group of the Pleiades was well observed here on January 30th ult. The sky was perfectly clear and atmospheric conditions quite good.

The following are the observed times of immersion of some of the brighter stars in Central Standard Time.

Star.	Immersion. Central Time.		
Electra	9h	28m	08s
Celoena	9	31	47
Taygeta, not occulted, passed just above the Moon's edge.			
Maia	10	17	39
Merope just cleared edge of limb.			
Triangle of small stars, companions of Alcyone:—			
a	11	00	10 ?
b	11	01	11
c	11	03	28
Alcyone	11	09	06

Instrument used, 4-inch telescope. DAVID E. HADDEN.

Alta, Iowa.

Lat. $+42^\circ\ 40'$
Long. $6^h\ 21^m$ } approx.

The Leonids.—Probably on account of the presence of the gibbous Moon which glided nearer and nearer each night to the radiant point the Leonids were not seen in unusual numbers in last November. During prolonged watches on successive nights they were only one fourth the number counted at the same hours of the night and morning in November 1896. The inference is that only those of sufficient size to outshine the atmospheric radiance were visible, the smaller bodies, which may have appeared in unusual numbers, passing unnoticed. Of those seen, several left lingering trains of luminosity, and a few accounts have come of a detonating meteor of great brilliancy passing over this city on the morning of Nov. 16th. ROSE O'HALLORAN.

Dec. 30.

The Leonid Display, 1897.—Cloudy weather interfered with observations of the meteoric shower of Leonids on November 13th and 14th. Our town night watchman reported to me that on November 15th at about 4:00 A. M. a fine display of shooting stars was observed in the vicinity of Leo, lasting about half an hour. DAVID E. HADDEN.

Alta, Iowa, Feb. 14, 1898.

The Moon.

	Phases.	Rises. (Central Standard time at Northfield; Local Time 13m less)			Sets.		
		h	m		h	m	
Mar. 8	Full Moon	6	58	P. M.	6	45	A. M.
15	Last Quarter	2	16	A. M.	10	44	"
22	New Moon	5	56	"	7	18	P. M.
30	First Quarter	10	0	"	2	12	A. M.

Occultations Visible at Washington.

Date 1898.	Star's Name.	Magnitude.	IMMERSION. Washington M. T.		Angle f'm N pt.	EMERSION. Washington M. T.		Angle f'm N pt.	Duration.	
			h	m	°	h	m	°	h	m
Mar. 3	85 Geminorum	6.0	15	6	158	15	44	246	0	38
4	δ Cancri	5.7	5	22	61	6	15	334	0	53
10	75 Virginis†	6.0	8	23	90	9	13	326	0	50
25	ε Arietis	4.6	6	6	161	6	17	177	0	11
26	27 Tauri	4.0	4	8	82	5	38	257	1	30
26	26 Tauri	7.0	4	9	130	5	6	208	0	57
26	28 Tauri	6.2	4	13	63	5	40	276	1	27

VARIABLE STARS.

I. A. PARKHURST.

Minima of the Variable Stars of the Algol Type.

[Given to the nearest hour in Greenwich Mean Time.]

1898.

ALGOL.

d	h
Apr. 10	15
13	12
16	9

U CEPHEI.

d	h
Apr. 1	18
6	17
11	17
16	17
21	16
26	16

R CANIS MAJORIS.

Every 8th min.
P = 1d 3h.3

d	h
Apr. 4	14
13	16
22	18

S CANCRI.

d	h
Apr. 3	
22	8

S VELORUM.

d	h
Apr. 6	7

δ LIBRAE.

d	h
Apr. 5	11
7	19
12	10
19	10
21	18
26	10
28	17

U CORONÆ.

d	h
Apr. 6	14
13	11
16	22
20	9
23	20
30	18

U OPHIUCHI.

d	h
Every 10th min.	
P = 20h.1	
Apr. 5	5
13	14
22	0
30	9

RS SAGITTARII.

d	h
Apr. 2	22
7	18
12	14
19	20
24	16

W DELPHINI.

d	h
Apr. 1	16
20	21
25	17

Y CYGNI.

d	h
Even minima.	
Apr. 2	14
5	14
8	13
11	13
14	13
17	13
20	13
23	13
26	13
29	13
Odd minima.	
Apr. 3	17
6	17
9	17
12	17
15	17
18	17
21	17
24	17
27	16
30	16

The ephemeris of the Algol stars is based on the elements given in Chandler's Third Catalogue, except that for Y Cygni I have used Duner's revised elements, given in *Astronomische Nachrichten* No. 3467. The times are taken from Hartwig's ephemeris in the Vierteljahrsschrift, except for S Velorum and RS Sagittarii, which Hartwig does not give, and Y Cygni. The long period variables are taken from Dr. Chandler's ephemeris in *Astronomical Journal* No. 420.

Maxima and Minima of Long Period Variables.

1898.

(From Dr. S. C. Chandler's ephemeris in *Ast. Jour.* No. 420).

MAXIMA.

		Mag.	Apr.
110	S Tucanæ	8.5	29:
146	T Sculptoris	8	1:
243	U Cassiopeæ	8.5	13
267	V Andromedæ	8	15
845	R Ceti	8	4
1717	V Tauri	9	25
2583	L_2 Puppis	3 5	13
4557	S Ursæ Majoris	7.5	3
5174	RS Virginis	8	10
5583	X Libræ	9 5	1
5675	V Coronæ	7.5	29
5795	W Scorpii	10 5	24
5830	R Scorpii	10	7
6062	RR Scorpii	7	14
6921	S Sagittarii	9.5	20
7120	χ Cygni	5	4
7444	T Delphini	9	7
7450	V Aquarii	8	20
7456	RR Cygni	8.5	16
8324	V Cassiopeæ	7.5	19
			May
62	S Sculptoris	6.5	12:
432	S Cassiopeæ	8	13
782	R Arietis	8	5
1805	V Orionis	8.5	28
1944	S Orionis	9	10
3060	U Cancri	9.5	22
3637	S Carinæ	6	29
4260	W Centauri	8.5	31:
4948	R Canum Ven.	6.5	26
5037	RR Virginis	11	17
5617	U Libræ	9	1
5704	RR Libræ	8 5	9
5831	S Scorpii	10	17
6682	X Ophiuchi	7	6
7118	X Aquilæ	8.5	20
7242	S Aquilæ	9	26
7260	Z Aquilæ	9	7:
7458	V Delphini	9	12
7560	R Vulpeculæ	8	15
7994	R Piscis austr.	8.5	20:

MINIMA.

		Mag.	Apr.
294	W Cassiopeæ	11.5	4:
1635	R Reticuli	< 13	—
2013	U Aurigæ	12	—
2780	T Geminorum	< 13	—
3128	R Pyxidis	< 11	—
3170	S Hydræ	< 12	—
3418	R Carinæ	9.5	30
5157	S Bootis	12.5	17
5338	U Bootis	12.5	26
5511	RS Libræ	13	—
5677	R Serpentis	13	17
6050	RS Scorpii	11.5	—
7155	RR Aquilæ	< 12	—
7234	R Capricorni	< 13	—
7482	U Pavonis	< 12	—
7577	X Capricorni	< 16	30:
8290	R Pegasi	< 13	20:
8512	R Aquarii	11	—
			May
103	T Andromedæ	13	22
268	X Sculptoris	12	—
1018	R Horologii	10	—
1113	U Arietis	< 11	—
1761	R Orionis	12	2:
1803	T Leporis	11	—
2445	W Monocer.	< 10	—
3425	X Hydræ	12	—
3567	V Leonis	< 13	—
4521	R Virginis	10.5	23
4573	RU Virginis	12	—
5438	Y Libræ	12	—
5494	S Libræ	< 13	—
5601	S Ursæ minoris	11.5	—
5776	X Scorpii	< 13	—
5856	W Ophiuchi	< 13	—
5950	W Herculis	13	1
6871	V Lyræ	< 12	—
7077	T Pavonis	12	—
7085	RT Cygni	10.5	29
7404	R Microscopii	12	8
7431	S Delphini	11	25
7455	U Capricorni	< 13	—
7468	T Aquarii	12.5	20
7733	Y Capricorni	14	—
8153	R Lacertæ	< 13	—

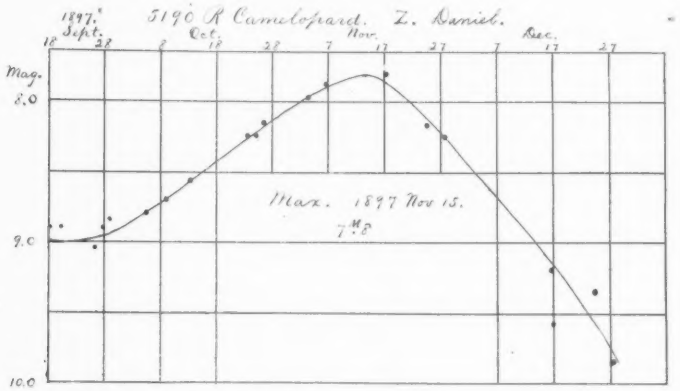
NEW EDITION OF THE DURCHMUSTERUNG CHARTS. The long-desired appearance of the new edition of the charts of the Northern Durchmusterung, bids fair soon to be realized. The work has been undertaken by the publishing house of A. Marcus and E. Weber, at Bonn, Germany, under the direction of Professor F. Küstner, the director of the Bonn Observatory, on the condition that orders for 100 sets at 70 marks each (about $17.50) are in their hands by May 1, 1898. The sets comprises 40 charts each 16½ by about 23 inches, besides margin, with a preface containing all the corrections known up to the end of the year 1897. The charts cover the sky from the north pole to 2° south declination, and are almost indispensable to observers of variable stars. It is hoped that orders will

be sent promptly to the publishers, so that the new edition may appear without delay.

MAXIMA OBSERVED BY MR. ZACCHEUS DANIEL, AT HAWTHORN, PENN.

				m	
3825 R Ursae Majoris,	1897	Nov.	22,	7.3	9 observations
6549 W Lyrae		Oct.	20,	8.2	7 "
5190 R Camelopardalis		Nov.	15,	7.8	19 "

The light curve for R Camelopardalis is given herewith. The date of maximum is 25 days later than Chandler's revised elements, but the star is known to be "somewhat irregular."



7792 *SS Cygni.* Following are 26 observations covering the last maximum, by Daniel, Sperra and Parkhurst.

	Gr. time.	m			Gr. time.	m	
1898 Jan.	5 50	11.30	P	Jan.	26.50	8.93	P
	8.48	11 3	D		27.54	9.00	P
	16.50	11.0	P		29.51	9 0	D
	17.50	8 65	D		29.52	9.00	P
	17.53	8.90	P		31.50	9 54	P
	18.50	8.7	D	Feb.	1.54	9.9	D
	18 50	8.7	S		2 50	10.36	P
	18 52	8 80	P		2.50	10 4	D
	19.00	8 68	P		2.96	10.7	D
	20 50	8 58	P		3.0	11±	S
	23.50	8.90	P		3.52	10 80	P
	24.5	8.7	S		3 97	11.2	D
	24.52	8.7	D		5 52	11.35	P

The rise was at the rate of at least 0.09m per hour and the star passed the 9.5m Jan. 17.2, remaining above the 9th magnitude 11 days, and above normal 19 days.[5] A smooth curve gives a maximum Jan. 20.5, at 8.6m.

In the last number of the *Vierteljahrsschrift* Dr. Hartwig deduces from the maxima of the year 1897 a mean period of 63 days, but to obtain this he has overlooked the maximum of April, and counted the two adjacent intervals as one, thus getting a mean period 10 days too long. The April maximum was observed at the Harvard College Observatory and by Mr. Sperra.

2815 *U GEMINORUM* passed a maximum about Feb. 3 or 4, just as the moon, nearly full, was so near as to make observations very difficult. However this is the fifth consecutive maximum about which there is no doubt, the mean interval being 86 days, agreeing with Chandler's period.

THE VARIABLE OF SHORTEST KNOWN PERIOD. Two important articles have appeared lately in regard to 8598 U Pegasi, which leave its title to the above distinction somewhat in doubt. (See POPULAR ASTRONOMY, Vol. III, pages 214 and 400 for charts etc.) In the Third Catalogue the period is given as $5^h\ 32^m\ 15^s$, with the minimum preceding the maximum by $3^h\ 14^m$. In H. C. O. Circular No. 23, printed elsewhere in this number, Professor Pickering finds the alternate minima of unequal brightness, 9.90^m and 9.75^m, with the period for the double variation $8^h\ 59^m\ 41^s$. Finally in *Astronomical Journal* No. 426, Dr. Chandler discusses this circular and his own observations, finding a period half that given by Pickering, $4^h\ 29^m\ 50^s.67$, with the alternate minima of the same brightness, $9^m.64$ (on a scale slightly different from Pickering's) and a "symmetrical light curve, with equal intervals of increase and decrease." The question of the comparative accuracy of photometric and eye-observations is also considered, and is very important in this connection. The star is too near the Sun for observation during March, but later in the season it is hoped that observers of some experience will give especial attention to the question of the comparative brightness of the alternate minima.

MARENGO, ILL., 1898, Feb. 8.

Harvard College Observatory Circular No. 24 announces the discovery of 27 new variables, 22 of them south of the equator, giving the spectral type and range of variation of most of them; also further results for a number of previously announced variables. Probably the most interesting for our readers will be the last in the list, in the constellation Cassiopeia. Its place is:

R. A. 23^h	55^m	53^s,	Dec. $+54°$	$52'$ 3.	(1855)
	58	10	55	7.3	(1900)

The spectrum is of the third type, as is usual with long period variables, and the range of variation, as found from 101 photographic plates, is from $9^m.8$ to less than $13^m.4$. This star is easily found, being 4 minutes (40′ of arc) following the 5th magnitude star *sigma* Cassiopeia. The accompanying chart,* on the scale the Durchmusterung, will aid in identifying the variable.

Omicron Ceti (Mira).—My first observation of Mira Ceti for the past apparition was made on October 24th, 1897, when the star was easily picked up with the naked eye, being a very little brighter than Nu Ceti. It has been observed on every available night up to the present time.

The following results represent my estimates of brightness in star magnitudes on the dates given. The observations were usually taken between the hours of 9 and 10 P. M. Central time both with the naked eye and 4-inch telescope and a few with a good field glass.

1897	Magnitude.	1897	Magnitude.
October 24	4.4	December 6	3.2
25	4.4	7	3.0
28	4.1	14	3.4
29	4.0	17	3.5
November 1	3.9	20	3.5
2	3.8	21	3.6

* This chart was lost while in the hands of the engraver. The chart will appear later

1897.	Magnitude.	1897.	Magnitude.
5	3.7	24	3.8
11	3.6	29	3.9
15	3.5	31	4.0
		1898	
16	3.5	January 2	4.0
17	3.5	7	4.1
18	3.5	9	4.3
19	3.4	15	4.5
20	3.3	16	4.5
22	3.3	21	5.5
28	3.4	23	5.5
		30	6.0

An inspection of the above indicates that maximum was not reached until about the 7th of December, fully 28 days later than the predicted date in the "Companion to the Observatory." Insufficient observations between November 28th and December 14th on account of unfavorable atmospheric conditions and bright moonlight near the date of maximum were unavoidable factors in determining the exact date and apparent magnitude of the "wonderful" star at its maximum phase. DAVID E. HADDEN.

Alta, Iowa, Feb. 14, 1898.

Observations of *o* Ceti in 1897.—The following abstract of 53 observations of this variable indicate that it attained a greater magnitude than during the three previous maxima, and that this occurred between Nov. 21st and Dec. 1st. To avoid repetition the nights of marked change only are mentioned.

Aug. 20th. Half a magnitude brighter than companion star.

Sept. 29th. Equal to adjacent star of 8th magnitude.

Oct. 15th. Two tenths brighter than 70 Ceti.

Oct. 17th. Equal to 66 Ceti; not as bright as ξ Piscium which it resembles in tint.

Oct. 23d. Nearly equal to ξ Piscium.

Oct. 26th. Brighter than ξ Piscium. A week of cloudy weather intervened.

Nov. 3d. Moonlight. Brighter than δ Ceti.

Nov. 10th. Moonlight. For the first time since the maxima have occurred out of sunlight, the variable is as bright as γ Ceti.

Nov. 21st. One fourth of a magnitude brighter than γ when near meridian on a dark sky.

Dec. 1st. One fourth brighter than γ Ceti. Moonlight. Clouds.

Dec. 12th. Equal to γ Ceti.

Dec. 14th. Not fully as bright as γ Ceti in a clear dark sky.

Dec. 16th. About one fourth of a magnitude dimmer than γ Ceti.

Dec. 21st. Midway between γ and δ Ceti.

Dec 26th. Scarcely one fourth of a magnitude brighter than δ Ceti.

Dec. 30th. Equal to δ Ceti. ROSE O'HALLORAN.

San Francisco, Dec. 31st, 1897.

COMET NOTES.

Definitive Determination of the Orbit of Comet 1892 II.—In *Astronomische Nachrichten* No. 3472 Dr. L. Steiner gives the results of his investigations for the orbit of the telescopic comet discovered by Mr. W. F. Denning, March 18, 1892. This comet though faint was visible for nearly a year, so

that Dr. Steiner was able to group his data into twelve normal places, at intervals of nearly a month. He finds that an hyperbolic path will represent the observations a little better than a parabola. The difference between the eccentricity derived and that of a parabola is, however, only about five times the probable error of the result. This is within the range of the possible effects of systematic errors upon the results, so that the evidence of the hyperbolic nature of the orbit is very slight. The following are the most probable elements as determined by Dr. Steiner:

$$T = 1892 \text{ May } 11.261935 \pm 0.003827 \text{ Berlin M. T.}$$

	°	′	″		
$\pi =$	22	45	42.40	}	
$\upsilon =$	253	25	50.92	}	1892.0
$i =$	89	41	54.10	}	

$$\log q = 0.2946197 \pm 0.0000079$$
$$e = 1.000345 \pm 0.000064$$

Ephemeris of Winnecke's Comet.

[Continued from p. 557, Feb. No.]

		R. A.			Decl.			log Δ
		h	m	s	°	′	″	
Mar.	1.5	19	47	58.61	− 14	26	33.5	
	3.5	19	58	33.08	− 14	29	51.8	0.1317
	5.5	20	9	6.50	− 14	31	21.6	
	7.5		19	37.83	− 14	31	3.7	0.1340
	9.5		30	6.29	− 14	28	58.1	
	11.5		40	31.13	− 14	25	6.6	0.1350
	13.5	20	50	51.58	− 14	19	30.1	
	15.5	21	1	6.99	− 14	12	12.0	0.1420
	17.5		11	16.69	− 14	3	14.8	
	19.5		21	19.97	− 13	52	43.2	0.1473
	21.5		31	16.28	− 13	40	41.0	
	23.5		41	5.15	− 13	27	13.4	0.1533
	25.5	21	50	46.01	− 13	12	24.9	
	27.5	22	0	18.56	− 12	56	20.8	0.1600
	29.5	22	9	42.26	− 12	39	6.8	
	31.5	22	18	57.29	− 12	20	49.4	0.1671

Re-discovery of Winnecke's Periodic Comet.—Winnecke's Periodic Comet was discovered on New Years night by the writer, the position being α 15^h 19^m 2^s.51, δ − 3° 58′ 34″.3 at 2^h 5^m 42^s Gr. M. T. of January 2.

The comet was thus 2^m east and about 9′ south of the place as predicted by Hillebrand.

It was extremely faint with the 36-inch refractor and very small—not over 10″ to 15″ diameter.

An observation on January 21^d 1^h 27^m 18^s Gr. M. T. gives its position as α 16^h 33^m 21^s.65, δ − 8° 26′ 30″.8.

It was considerably brighter than at re-discovery and larger.

Lick Observatory, University of California. C. D. PERRINE.
Jan. 24, 1898.

GENERAL NOTES.

With this issue we begin the new volume. The renewed subscription list already in hand is encouragingly large, and if it continues in the same way we will try some improvements which will meet the known wants of our readers in some respects. This journal will be issued every month during the year 1898.

The size of this number is much increased in order to admit some mathematical articles that would interest a considerable portion of our readers. Some new series of popular articles will begin in our next.

Observations of the Milky Way.—The three useful articles, on further study of the Galaxy, published consecutively in the last three issues of this journal have awakened in the minds of amateurs an interest in this line of work that promises well. The first article was on the necessity of further researches on the Milky Way. The second was about new charts for inserting the sketches of the Milky Way, and the third was the stellar bands in the Zodiac from Gemini to Scorpio. The charts for inserting sketches have been received. The first covers that portion of the Galaxy from Aquila to Cassiopeia; the second, from Casseopeia to Monoceros. These charts are well engraved and are furnished upon their margins with ample references, so that the stars in the body of the charts may be easily and certainly identified. This inviting line of work ought to find many earnest students. The author, Mr. A. Pannekoek, has planned the work well, and the instructions for doing it are careful and explicit. The charts for inserting the sketches will be furnished to any who desire to use them for such a purpose, free of charge.

Miss O'Halloran's interesting notes given elsewhere should have appeared in our last number. It was our oversight that they were omitted.

New Edition of Argelander's Atlas —Messrs. A. Marcus and E. Weber, publishers in Bonn, Germany, announce that they are planning to issue a new edition of that most valuable work published by Argelander in 1863.

Atlas des nordlichen gestirnten Himmels für den Anfang des Jahres 1855 unter Mitwirkung der Herren Professor Dr. E. Schönfeld und Professor Dr. A. Krüger nach der in den Jahren 1852 bis 1862 auf der Königlichen Universitäts-Sternwarte zu Bonn durchgeführten Durchmusterung des nördlichen Himmels entworfen und in Namen der Sternwarte herausgegeben von dem Director derselben. Text gr. Folio und 40 Karten grösstes Karten-Format. 1863. 90M.

The publishers propose to insert the corrections of all the known errors up to the close of 1897. Owing to the difficulty and expense attending the work the publishers must be assured by the first of May, 1898, that at least 100 copies of the Atlas will be purchased of them.

The new plates will be reproduced by the photolithographic process.

All copies, orders for which are in the hands of the publishers before May 1, 1898, provided at least 100 are ordered, will be furnished at the price of 70 marks per copy. Single sheets will be furnished at 2 marks apiece, if ordered at the same time with a complete copy.

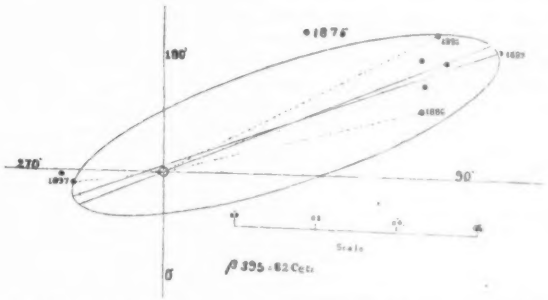
After May 1, 1898, the price will be raised to at least 120 marks per copy and single sheets will no longer be furnished.

The work will be done under the supervision of Professor F. Küstner, Director of the Royal Observatory at Bonn, so that astronomers may feel assured that it will be done in a satisfactory manner and that the plates of the atlas will be as free as possible from errors.

The price of 70M = $16.87 is certainly so small that a great many astronomers of this country will gladly avail themselves of the opportunity to obtain this magnificent standard work. W.

Orbit of the New Binary β 395 = 82 Ceti —An intensely interesting article by Dr. T. J. J. See appeared some time ago in No. 3455 of the *Astronomische Nachrichten*, relating to the orbit of the new binary 395 which is the same as 82 Ceti. This double star was noticed by Dr See on the morning of Aug. 18, 1897, while sweeping over the constellation of Cetus. With a power of 500 the star was elongated, but it required a power of 1500 to divide the star and well define the images of the components, which were of 6.5 and of 6 6 magnitude respectively, and both yellow.

This star was discovered by Mr. Burnham, of Chicago, in 1875 with his noble six-inch telescope. It has since been measured by several prominent double-star observers, including Hall, Leavenworth, Burnham and See. Since 1875 the double has made about one and one-half revolutions which seems to have escaped the notice of observers until its more thorough study was undertaken by Dr. See late last year.



The cut of its orbit is given here as it appeared in *A. N.* cited above. The period of revolution of this interesting double star is 16.3 year. Only three known telescopic binaries have shorter periods. They are β 883, χ Pegasi and δ Equalei, with periods of 5.5, 11.42, and 11.45 years respectively.

Photographic Magnitudes.—In determining the photographic magnitudes of the stars it is a matter of great importance to know how much their relative brightness will vary on different plates, or on different portions of the same plate. It is especially important to determine the amount of this error since it is not easily eliminated and has been supposed to be large by some persons not familiar with stellar photographs. A moment's investigation of photographs of the same portion of the sky shows that this source of error is small, so small that it is not readily determined by direct measurement. The uniformity of different portions of the film is shown by allowing the star to trail over the plate. The different portions of the trails appear equally intense, and no variation is perceptible to the eye. A much more delicate test was found in the discussion of a series of measures, made by Miss. E. F. Leland, of the variables discovered by Professor Bailey in the Cluster Messier 5. Sixty-three of these variables were compared on 41 plates by Argelander's method, with a sequence of comparison stars. Estimates were made of the difference in grades of each variable from the next brighter and the next fainter star of the sequence. The sum of these differences gives the interval between the comparison stars and combining all the results gives, in general, several measures of each interval on each plate. Each comparison star in turn may then be regarded as a variable and its changes in

light determined from the next brighter and next fainter star of the sequence. Comparatively few measures were made of the six brightest and the three faintest stars of the sequence. The five intermediate stars were measured on 41, 39, 38, 30 and 30 photographs respectively. The corresponding ranges in the measures derived from each plate, none being rejected, were 0.14, 0.10, 0.12, 0 15, and 0.08 magnitudes, and the average deviations, 0.02, 0 01, 0.02, 0.03 and 0.02. The largest residual was 0.10 and this depended upon two estimates only. We find, therefore, that on the average, five stars were measured on 35 plates with a range of 0.12 magnitudes and an average deviation of 0.02 magnitudes. The total number of estimates from which these results are derived is 4294. The average deviation 0.02 includes: 1st. The errors of observation which are increased by the fact that four estimates enter into each determination, but are diminished since on the average twelve determinations were made of each interval on each plate. 2d. Errors due to neglecting hundredths of a magnitude, the computation so far being made only to tenths. 3d. Errors due to irregularities in the film which enter with their full value into the result. Since the combined effect of these three sources of error is only ± 0 02, it is evident that neither of them can be large. The errors due to the film are in fact so small that there is no evidence that they exist and more delicate methods of measurements are required to render them perceptible.—Harvard College Observatory, Circular No. 22.

January 4, 1898. EDWARD C. PICKERING.

Polarizing Photometers.—Nearly all of the photometric measurements obtained at the Harvard College Observatory during the last twenty years have been made with modifications of three forms of photometers which are identical in principal. The first of these is described in the Annals, Vol. XI, Part I, and was used for the observations contained in that publication. The second, the meridian photometer, furnished the observations contained in the Annals, Volumes XIV, XXIII, XXIV, and XXXIV. The third photometer is described in the *Astrophysical Journal*, Vol II, p. 89. In all of these instruments the star to be measured is compared directly with another star by means of a double image prism and Nicol. In the first instrument, the images of two adjacent stars are brought together by a double image prism; in the second, images of two stars however distant are brought together by reflecting them by prisms or mirrors into two object glasses; in the third photometer, images formed by a large telescope, of two stars not more than half a degree apart, are brought together by achromatic prisms.

An objection to the first form of photometer is that the emergent pencils of the images compared do not coincide. Small errors may therefore be introduced by irregularities in the corner of the eye of the observer, or if he holds his eye in such a position that a portion of one image will be cut off by the edge of the pupil. This difficulty has recently been remedied by placing a second double image prism in the focal plane of the telescope, so that it does not affect the position of the two images but makes the emergent pencils coincide. A surprising degree of accuracy may then be obtained in the measures. Comparisons of the star *o* Ceti with the adjacent star—3° 355, which follows it about 10ˢ, have been made by Mr. O. C. Wendell with the 15 inch equatorial of this Observatory, on 191 nights during the last five years. Until recently, the observations were made with the first form of photometer, the emergent pencils overlapping by about two fifths of their diameter. Generally thirty-two settings were made each night, and the mean of the differences of each pair of sets of four settings each on the first 14 nights of obser-

vation made during the present opposition was ± 0.074 magnitudes. On the last five nights the pencils were made to coincide, as just described, and the corresponding average differences were 0.020, 0.002, 0.025, 0.022 and 0.032, mean ± 0.020. On the second of these nights, January 19, 1898, the eight measures of the difference in light of the two stars, each derived from four settings, were 4.48, 4.48, 4.48, 4.48, 4.48, 4.48, 4.48 and 4.47. Owing to the variations in the transparency of the air in different parts of the sky, this degree of accordance can only be expected when stars near together are compared.

The accuracy of the results attainable with the third form of photometer is shown in the *Astrophysical Journ.* Vol. III, p. 281, and in the observations of U Pegasi described in Circular No. 23. Mr. Chandler (*Astron. Journ.* Vol. XVIII, p. 140) while admitting the principal conclusions given in that circular, denies the reality of the small difference 0.15 between the primary and secondary minima. This quantity is so small that it doubtless could only be surely determined by the most accurate photometric measurements. The individual results derived from Mr. Wendell's observations are therefore given below. Twelve observations, each consisting of 16 settings were made when the star was within twenty minutes of its primary minimum. Deriving from each of these, by means of the light curve, the magnitude of this minimum we obtain on October 18, 1897, 9.89, 9.94 and 9.96; on December 30, 9.90, 9.95 and 9.93; on January 1, 1898, 9.93, 9.86 and 9.85; on January 5, 9.85 and on January 7, 9.86 and 9.88. Mean of all, 9.90, greatest value 9.96, least value 9.85, average deviation ± 0.035. Similarly, fourteen observations were taken within twenty minutes of the secondary minimum with the results on October 18, 1897, 9.75 and 9.71; on October 29, 9.74, 9.69, 9.70 and 9.70; on December 28, 9.78, 9.77, 9.76 and 9.80; on January 3, 1898, 9.77, 9.77, 9.74 and 9.78. Mean of all, 9.75, greatest value 9.80, least value 9.69, average deviation ± 0.029. It will be noticed that the largest value of the secondary minimum is 0.05 less than the smallest value of the primary minimum. If we assume that the primary and secondary minima are really equal, all of the first of these values give positive residuals with the mean value + 0.064 and all of the second (with one exception + 0.01) give negative values with the mean value — 0.070. The probability that the two minima are really equal and that these deviations are due to accidental error is extremely small. It is the same as that a person should draw a red card from a pack of cards twelve times in succession, and then should draw a black card thirteen times out of fourteen. On the other hand, if these deviations are due to a systematic error, it is very singular that this error always has one value at the time of a principal minimum, and another at the time of a secondary minimum.

EDWARD C. PICKERING.

Harvard College Observatory, Circular No. 25.
February 8, 1898.

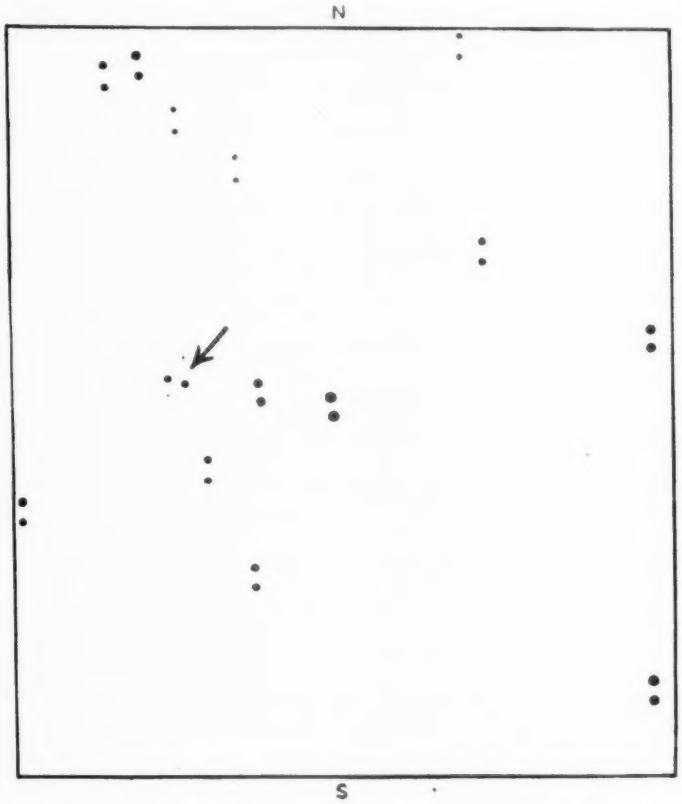
A New "Run Away" Star.—Professor J. C. Kapteyn in the *Astronomische Nachrichten* No. 3465 announces the discovery of a star having a greater proper motion than any hitherto known.

"The star Cordoba Zone Catalogue $5^h.243$ has a proper motion of 8″.7 in a great circle, as derived from the following observations:

	Magn.	Epoch.	α 1875			δ 1875	
			h	m	s	°	′
Cord. Z. C. (2 obs.)	8	1873.04	5	6	40.61	−44	58.176
Cape Photographic D.M.	9.2	1890.1			50.8		59.9
Cape Catalogue Plate	—	1893 9			53.8		60.2
Innes, Equatorial	8.2	1897.1			55.8		60.4
Cape Meridian Obs.	—	1897.81			56.0		60.530

These are all well represented by a proper motion of $+ 0^s.621$ in right ascension, and of $- 5''.70$ in declination. Innes finds the star to be orange yellow."

The motion of this star exceeds, by over one and one-half seconds of arc, that of the so-called "run away" star Groombridge 1830. The star is much fainter than Groombrige 1830, and it will now be an interesting problem to determine its parallax and thus ascertain whether its real movement compares with its apparent motion.



CORDOBA Z. C. $5^h.243$ SHOWING PROPER MOTION GREATEST KNOWN.

The accompanying cut was made from a tracing from a photograph kindly sent us by Professor Pickering. The photograph was "enlarged nine times from

two negatives taken with the 8-inch Bache telescope at the Arequipa Station of the Harvard College Observatory. The scale of the original photographs is $180'' = 0.1\ cm$. The plates were superposed so that the images of the stars on one should be a short distance below those on the other. The motion of *Z. C.* 5^h, 243, which is indicated by an arrow, is at once apparent from the displacement of the line connecting the two photographic images of this object. The southern of each pair of images and the right hand image of 5^h, 243 are reproduced from a photograph taken October 8, 1889, with an exposure of 14^m. The northern images are reproduced from a photograph taken November 10, 1896, with an exposure of 12^m."

BOOK NOTICE.

A New Astronomy For Beginners is the title of a new book by Professor D. P. Todd of Amherst College and published by the American Book Company of Chicago and New York. In size it is 12 mo, 480 pages, with five colored plates and otherwise very fully and well illustrated. Price $1.30.

"How to teach and how to study the elements of astronomy" is a theme that ought to engage the attention of teachers more fully and more carefully than appears to have been the fact in the past. Since the meeting of the "committees of ten" at the Chicago University some three years ago to formulate entrance conditions to college, more attention has been given to the methods of teaching science than before. In the discussions of those committees some things were very plainly shown, two of which were very evident to us in regard to the views of one committee at least, and that one was had in charge of the branches of physics and astronomy. That the laboratory methods of teaching physics had already gained a strong hold, by actual use, in the minds of eight members of the committee was at once evident; that a large part of the committee had taught little or no astronomy and therefore had very little to say about it, and apparently less interest in the branch than almost any other—so much so as to question its place in the high school or preparatory school course and at the same time to claim a whole year in these preparatory schools for the elements of physics. This was a surprising attitude for a body of representative teachers to assume. It was unjust to the kindred branch of astronomy and indicated too strong a bent to the special line of work in which they were most interested. The real question of what might be best for the schools of the land seems to have a second place in thought and plan.

This much is said to introduce one of the leading features of Professor Todd's new book, and that is the idea of laboratory teaching in connection with the study of the elements of astronomy. We think he has presented the matter in this new book in this regard strongly and well. He has done it by the aid of ingenious diagrams and common place illustrations that ought to reach the minds of students and readers easily and effectively. We think the interest of those pursuing this work will be sustained and heightened by the fact that the author has written everything to date, showing pretty fully and well what are the lines of thought and activity among astronomers at the present time in the wide range of the study. To do this part of the work well, and write it down to the comprehension of young students is no easy task. Those things in the history of the subject which are definitely known are easy to state, but to present the new and latest phases of knowledge and discovery in such a way as to represent the best judgment in professional scholarship is a difficult thing to do, especially if an

author is writing for young readers or those having little experience in the study of science.

Since our opinion was asked a few weeks ago concerning the merits of this new book, we have been much interested in seeing how students in a neighboring high school could use it in the class room and in individual study We are not surprised to find favorable results as the outcome so far although but little advancement has yet been made.

This new book is commended to the attention of our readers generally.

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